Entanglement Detection Criterion in terms of Probability Amplitudes

Sushamana Sharma[1,*], Rakesh Kothari[2] and Abhijit Kulshrestha[3]
[1,2,3] Dept. of Physics, Jodhpur Institute of Engineering & Technology, Jodhpur, Rajasthan, India.
*sushmana.sharma@jietjodhpur.com

Entanglement has widely researched nowadays worldwide because of its applications in the field of technology. A significant problem is its detection is dealt in this paper. Many methods for entanglement detection and problems associated them are given. An entanglement criterion based on probability amplitudes is derived and tricks which helps in finding a suitable entanglement witness for two qubit systems is explained.

Keywords: Entaglement, Probability amplitude, Two qubit system.

1. INTRODUCTION

Quantum mechanics is an immensely successful theory, occupying a unique position in the history of science. It finds wide ranging applications from macroscopic superconductivity to microscopic theory of elementary particles; quantum information science [1,2]. Entanglement is one of the features of quantum mechanics most at odds with a classical world view [3]. The entangled states are those states which cannot be written as a convex combination of product state [4]. Pure entangled states are those having maximum entanglement and examples are singlet state and Bell states. For bipartite system having subsystems 1 and 2, it has the following forms [3]:

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

The density matrix of mixed state is [5], known as Werner states

$$\hat{\rho} = x|\psi\rangle\langle\psi| + (1-x)\frac{I}{4}$$

If the states are not maximally entangled then it does not allow for faithful teleportation and faster computation [6]. We need to quantify entanglement present in a given state as well as some tools to test whether the given state is entangled or not. For detection, the first test could be its definition which is not an easy test. The second test could be the violation of Bell's inequality [7], but not all entangled states violate Bell inequalities e.g. Werner states. Although it is not a good test, but it was the first test for detection of entanglement in laboratory [8]. Von Neumann entropy which may be expressed in terms

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of eigenvalues of the state is helpful in detection of entanglement in states [9]. For bipartite system, the well established and easy criterion is positive partial transpose (PPT) criterion which is given by A. Peres [10]. According to it, a state is said to be entangled if its partial transposed matrix is not a valid matrix. For a density operator to be a valid $\hat{\rho}^{pt}$, it must heve $Tr(\hat{\rho}^{pt}) = 1,\ det(\hat{\rho}^{pt}) \geq 0$ or having all non negative eigenvalues, $Tr(\hat{\rho}^{pt}\hat{\rho}') \geq 0$ for any density matrix $\hat{\rho}^{'}$ [11,12]. For a set of mixed bipartite state, if it's partially transposed matrix with respect to second particle, with element $\langle n_1 n_2|\hat{\rho}^{pt}|m_1 m_2\rangle \equiv \langle n_1 m_2|\hat{\rho}|m_1 n_2\rangle$

2. FORMULATION

A system of two qubits can always be expressed as [13]

$$|\psi\rangle = p|\psi_e\rangle + \sqrt{1-p^2}e^{i\theta}|\psi_f\rangle,$$

With $|\psi_e\rangle$ maximally entangled state and $|\psi_f\rangle$ factorizable state and if the phase angle θ is chosen random then its density matrix will be

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\hat{\rho} = p^2|\psi_e\rangle\langle\psi_e| + (1-p^2)|\psi_f\rangle\langle\psi_f|$$

$$\hat{\rho} = x|\psi_e\rangle\langle\psi_e| + (1-x)|\psi_f\rangle\langle\psi_f|\ ,$$

here$\mathrm{x} = \mathrm{p}^2$.

If $|\psi_e\rangle = |\phi^+\rangle$ and let the factorizable part $|\psi_f\rangle = |\chi\rangle$ which has the structure,

$$|\chi\rangle = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

Where $a, b, c\ and\ d$ are real amplitudes, for convenience let $a^2 + b^2 = 1;\ c^2 + d^2 = 1.$

Then the density matrix of the state under consideration will be

$$\hat{\rho} = x|\psi_e\rangle\langle\psi_e| + (1-x)|\chi\rangle\langle\chi| \tag{1}$$

Now transform the above matrix as under,

$$\hat{\rho}' = U\hat{\rho}_e U^{\dagger}$$

here $U = \sigma_z \otimes \sigma_x$. Calculating $Tr(\hat{\rho}^{pt}\hat{\rho}')$ with $\hat{\rho}^{pt}$ (the partial transpose of the matrix given in (1))

$$Tr(\hat{\rho}^{pt}\hat{\rho}') = \frac{1}{2}[(1-x)(ad-bc)^2 - x] \geq 0$$

$$Tr(\hat{\rho}^{pt}\hat{\rho}') = 0 = \frac{1}{2}[(1-x)(ad-bc)^2 - x],$$

or

$$x = \frac{(ad-bc)^2}{(ad-bc)^2+1}$$

and for negative value of $Tr(\hat{\rho}^{pt}\hat{\rho}')$,

$$x > \frac{(ad-bc)^2}{(ad-bc)^2+1} \qquad (2)$$

Example 1: For Bell state $|\psi_e\rangle = |\phi^{\pm}\rangle$, $\hat{\rho}_e = |\psi_e\rangle\langle\psi_e|$ and $|\psi_f\rangle = 0$

$$a = 0 = b = c = d$$

Putting in (2) gives $Tr(\hat{\rho}^{pt}\hat{\rho}') \geq 0$ for all values of $x > 0$.

Example 2: Werner state

Let the entangled part be $|\psi_e\rangle = |\psi^{\pm}\rangle$ and factorisable part $|\psi_f\rangle = (1-x)\frac{I}{4}$, using (2),

$$Tr(\hat{\rho}^{pt}\hat{\rho}') = \frac{(1-3p)}{4} \Rightarrow \begin{cases} \geq 0 & if x \leq 1/3 \\ < 0 & if x > 1/3 \end{cases}$$

If $\boldsymbol{x = 0}$ then state will be separable as it is evident from (2).

3. CONCLUSION

A condition is derived by which entanglement may be detected in two qubit systems. Though it is not universal witness but it is good upto a limit. The results match with the established results.

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