Effect of Coulomb Blockade on Josephson Super Current Across Superconductor/Quantum Dot/Superconductor Nano Junction

A. Dhyani, P. Dua, A.K. Chawla and B.S. Tewari*
Department of Physics, University of Petroleum and Energy Studies, Dehradun, Uttarakhand, India.
E-mail:-*bstewari@ddn.upes.ac.in*

The present paper deals with the study of Josephson supercurrent across the correlated single level quantum dot sandwiched between two s-wave superconductors. The renormalized Anderson model is used that includes in the Hamiltonian contribution corresponding to the correlated superconducting quantum dot state, effective BCS type attractive interaction in the superconducting leads, the electron tunneling energy term responsible for coupling of superconducting leads with the dot energy level thatrepresents the tunneling of Cooper pair between two superconductors and on-dot coulomb energy to take care of electronic correlations. The Green's function equations of motion technique are employed within the framework of Ambegaokar-Baratoff approach to analyze the Josephson supercurrent across such junction. It is pointed out that for large value of on-dot Coulomb energy, the Josephson supercurrent across superconducting Quantum dot (S-QD-S) junction decreases.

Keywords: Josephson supercurrent, Quantum dot, Electron tunneling, Cooper pair, Superconductivity, Nanoscopic junction.

1. INTRODUCTION

In recent years, charge transport in nanoscopic hybrid systems of quantum dot has been studied very widely due to rapid progress in nanoscale fabrication techniques. These studies open the possibility of the development of quantum computers and other advanced nano electronic devices. These hybrid structures consists a combination of quantum dot and connecting electrodes. There exists variety of combination depending on number of quantum dots sandwiched between different types of electrodes. Out of these possible combinations, S-Qd-S nanoscopic junction, in which a quantum dot sandwiched between two superconducting leads (electrodes), has been widely researched to study the charge transport through quantum dot [1-11].

The charge transport in S-QD-S junction depends on either single electron tunneling or Cooper pair tunneling from one superconductor to other superconductor through quantum dot and is generally directed by two processes. At low temperatures ($T<T_c$), the electronic transport is due to single particle tunneling process. In this process, Cooper pair from one superconducting lead breaks at the junction interface followed by tunneling of single electron to the other side and vice-versa through the discrete energy level of quantum dot. When the coherence length of the connected superconductor is larger than

the size of the quantum dot junction, there is possibility of tunneling of Cooper pair from one side of superconductor to other side superconductor through quantum dot without any pair breaking effect and this process leads to Josephson Cooper pair tunneling in S-QD-S junction [12-16]. Under such circumstances, the Andreev reflection: a process in which an incoming electron from the quantum dot reflects a hole and transferring a Cooper pair into the other superconducting lead, dominates over the Kondo effect and influence the transport behavior of the S-QD-S junction at low temperatures [17-18].

The Josephson supercurrent across S-QD-S junction mainly depends on the discrete energy spectrum of the quantum dot and the nature of superconducting gap in superconducting leads connected to it. It has been observed that Josephson supercurrent across such tunnel junctions depend on quantum dot level energy, coupling parameter of dot states with superconducting leads and Josephson Cooper pair tunneling. In these studies, a sharp resonance peak in Josephson supercurrent has also been predicted when QD state energy matches with the Fermi energy of S-QD-S junction. Inspite of the several theoretical attempts [17-18], the role of on-dot coulomb interaction on the QD state sandwiched between the superconducting leads on the Josephson supercurrent is not clearly understood so far. For a correlated QD where charging energy of the QD is large in comparison to the coupling energy of the QD to the leads, the Josephson supercurrent across the junction is dominated by single particle tunneling and the Kondo effect [19]. For a correlated dot due to strong on-dot Coulomb interaction, the double occupancy on the dot level becomes unlikely and under such conditions the single particle tunneling dominates over the bound electron tunneling in the S-QD-S junction [19-20]. Therefore, in the light of above facts, we have planned to analyze the role of on-dot Coulomb energy on the Josephson supercurrent across the S-QD-S tunnel junction. Further, it is assumed that superconducting-QD–Josephson junction is at low temperatures (i.e. $T<T_c$) and under such conditions, a theoretical analysis of the Josephson supercurrent within Ambegaokar and Baratoff [4-6,15] approach has been attempted.

2. THEORETICAL FORMULATION

As described above, there are experimental evidence of the large on dot Coulomb energy (of the order 1-2 eV) in QD and this energy play a dominant role on the electronic transport through S-QD-S junction. Therefore, in present section we analyze the effect of on dot electron-electron correlation on the Josephson supercurrent across the S-QD-S junction. For such purpose we used renormalized Anderson model coupled with reduced BCS Hamiltonian for superconducting leads suitable for such S-QD-S junction and may be described as follows:

$$H = H_D + \sum_{\eta=1,2} (H_\eta + H_{tunn,\ \eta}) \tag{1}$$

Where,

$$H_D = \sum_\sigma \varepsilon\, d_\sigma^+ d_\sigma + U' n_\uparrow^d n_\downarrow^d \tag{2}$$

$$H_{\eta} = \sum_{k\sigma} \varepsilon_k c^{+}_{\eta k\sigma} c_{\eta k\sigma} - \sum_k (\Delta_{\eta} c^{+}_{\eta k\uparrow} c^{+}_{\eta -k\downarrow} + \Delta^{+}_{\eta} c_{\eta -k\downarrow} c_{\eta k\uparrow}) \tag{3}$$

$$H_{tunn,\,\eta} = V \sum_{k\sigma} (c^{+}_{2k\sigma} d_{\sigma} + d^{+}_{\sigma} c_{1k\sigma} + c^{+}_{1k\sigma} d_{\sigma} + d^{+}_{\sigma} c_{2k\sigma}) \tag{4}$$

Where, H_D (Eq. 2) is the Hamiltonian for correlated QD which contain the single energy level (ε) which has $n^{d}_{\uparrow} = n^{d}_{\downarrow} = \frac{n}{2}$, where n be an integral spin. In (Eq. 2), U′ is the on dot Coulomb energy or Coulomb blockade energy. H_{η} (Eq. 3) is the BCS Hamiltonian for left (η=1) and right (η=2) side superconductor. In H_{η} the first term is the kinetic energy and second term represents the attractive interaction between the electrons of superconducting lead responsible to form Cooper pairs and formation of BCS superconducting state described by a gap at the Fermi level. $H_{tunn,\eta}$ (Eq. 4) represents the possibility of the single particle tunneling between left superconducting leads and QD state and vice-versa. The $c_{\eta k\sigma}\,(c^{+}_{\eta k\sigma})$ represents the annihilation (creation) operators for the superconducting lead and $d_{\sigma}\,(d^{+}_{\sigma})$ represents the annihilation (creation) operators for the dot state and U′ is the on dot Coulomb energy.

In order to study the electronic transport behavior of S-QD-S junction, we have employed the Green's function equation of motion technique [4,5]. Finally for our model Hamiltonian, we obtain the following Green's function:

$$\langle\langle c^{+}_{1-k\downarrow}; c^{+}_{1k\uparrow} \rangle\rangle = \frac{\Delta^{+}}{2\pi} \left[\frac{(\omega^2 - \tilde{\varepsilon}^2)(\omega^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\omega^2 + V^2\omega(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^2}{(\omega^2 - \alpha^2)(\omega^2 - \beta^2)(\omega^2 - \gamma^2)} \right] \tag{5}$$

where, $\tilde{\varepsilon} = \varepsilon + U'(n/2)$ taking $n^{d}_{\uparrow} = n^{d}_{\downarrow} = (n/2)$, a paramagnetic situation.

Here, we assume that both superconductors are identical and have same superconducting order parameter (i.e. $\Delta_1 = \Delta_2 = \Delta$) and also the coupling energy of the superconductor and dot states are identical.

The simple algebra leads the above expression (5) in the following form:

$$\langle\langle c^{+}_{1-k\downarrow}; c^{+}_{1k\uparrow} \rangle\rangle = \frac{I}{(\omega - \alpha)} + \frac{J}{(\omega + \alpha)} + \frac{K}{(\omega - \beta)} + \frac{L}{(\omega + \beta)} + \frac{M}{(\omega - \gamma)} + \frac{N}{(\omega + \gamma)}$$

where, $\alpha = \sqrt{\varepsilon_k^2 + \Delta^2}$

$$\beta = \sqrt{\frac{(\varepsilon_k^2 + \Delta^2 + \tilde{\varepsilon}^2 + 4V^2) + \sqrt{(\varepsilon_k^2 + \Delta^2 - \tilde{\varepsilon}^2)^2 + 8V^2\{(\varepsilon_k + \tilde{\varepsilon})^2 + \Delta^2\}}}{2}}$$

$$\gamma = \sqrt{\frac{(\varepsilon_k^2 + \Delta^2 + \tilde{\varepsilon}^2 + 4V^2) - \sqrt{(\varepsilon_k^2 + \Delta^2 - \tilde{\varepsilon}^2)^2 + 8V^2\{(\varepsilon_k + \tilde{\varepsilon})^2 + \Delta^2\}}}{2}}$$

$$I = \frac{(\alpha^2 - \tilde{\varepsilon}^2)(\alpha^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\alpha^2 + V^2\alpha(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^4}{2\alpha(\alpha^2 - \beta^2)(\alpha^2 - \gamma^2)}$$

$$J = -\frac{(\alpha^2 - \tilde{\varepsilon}^2)(\alpha^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\alpha^2 - V^2\alpha(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^4}{2\alpha(\alpha^2 - \beta^2)(\alpha^2 - \gamma^2)}$$

$$K = \frac{(\beta^2 - \tilde{\varepsilon}^2)(\beta^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\beta^2 + V^2\beta(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^4}{2\beta(\beta^2 - \alpha^2)(\beta^2 - \gamma^2)}$$

$$L = -\frac{(\beta^2 - \tilde{\varepsilon}^2)(\beta^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\beta^2 - V^2\beta(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^4}{2\beta(\beta^2 - \alpha^2)(\beta^2 - \gamma^2)}$$

$$M = \frac{(\gamma^2 - \tilde{\varepsilon}^2)(\gamma^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\gamma^2 + V^2\gamma(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^4}{2\gamma(\gamma^2 - \beta^2)(\gamma^2 - \alpha^2)}$$

$$N = -\frac{(\gamma^2 - \tilde{\varepsilon}^2)(\gamma^2 - \varepsilon_k^2 - \Delta^2) - 3V^2\gamma^2 - V^2\gamma(\tilde{\varepsilon} + \varepsilon_k) - 3V^2\tilde{\varepsilon}\varepsilon_k + 4V^4}{2\gamma(\gamma^2 - \beta^2)(\gamma^2 - \alpha^2)}$$

The superconducting order parameter can be obtained from the corresponding Green's function with the help of standard procedure [4-5,21]. Finally, we obtain the expression for superconducting order parameter for (S-QD-S) junction having a single level correlated QD as:

$$\Delta^+ = -\frac{\Delta^+}{N}\sum_k\left[\frac{I}{e^{\beta'\alpha}+1} + \frac{J}{e^{-\beta'\alpha}+1} + \frac{K}{e^{\beta'\beta}+1} + \frac{L}{e^{-\beta'\beta}+1} + \frac{M}{e^{\beta'\gamma}+1} + \frac{N}{e^{-\beta'\gamma}+1}\right] \quad (6)$$

where, $\beta' = 1/(k_B T)$.

Using above equation (6), one can estimate superconducting order parameter numerically in a self-consistent way by replacing summation over k value by an integral with cut-off energy and a constant density of states around the Fermi level [4,5]. From equation (6), it is clear that the superconducting order parameter depends on the temperature, Coulomb blockade energy and various parameters of the model Hamiltonian. Therefore to analyze the Josephson supercurrent through S-QD-S junction, we use the Ambegaokar-Baratoff formalism [4,5,11,21-23] which connects the superconducting order parameter with the Josephson supercurrent as:

$$I_c R_n = \frac{\pi\Delta(T)}{2e}\tanh\left\{\frac{\Delta(T)}{2k_B T}\right\} \tag{7}$$

Where I_c is the Josephson supercurrent and R_n is the junction resistance in the normal state. At T=0K, the above equation reduces in the following form:

$$I_{c0} R_n = \frac{\pi\Delta(0)}{2e} \tag{8}$$

Where, Δ(0) and Δ(T) are superconducting order parameter at T=0 and finite temperature T (<T_c). One can obtain from equation (7) at T=0K. Using equations (7) and (8), the renormalized Josephson supercurrent can be expressed as:

$$\frac{I_c}{I_{c0}} = \frac{\Delta(T)}{\Delta(0)}\tanh\left\{\frac{\Delta(T)}{2k_B T}\right\} \tag{9}$$

Using above equation (9) one can analyze the supercurrent across the S-QD-S junction.

3. RESULTS AND DISCUSSION

On performing numerical computation using the equation (6) and (9), we have analyzed in this section, the effect of the Coulomb blockade effect, dot energy level and temperature on the Josephson supercurrent across the S-QD-S junction. As a first step in Figure 1, we have plotted the renormalized Josephson supercurrent (I_c/I_{c0}) versus T/T_c for different values of the on dot Coulomb energy, keeping other parameters (V=0.01eV, ω_c =0.025eV, ε=0.001eV and n=1) fixed. One can observe from Figure 1 that as the Coulomb blockade energy increases the renormalized Josephson supercurrent decreases due to the preventing of Cooper pair tunneling across the junction near critical temperature. As the charging energy of the QD increases, the probability of the tunneling across the junction decreases results in the reduction of order parameter of the junction.

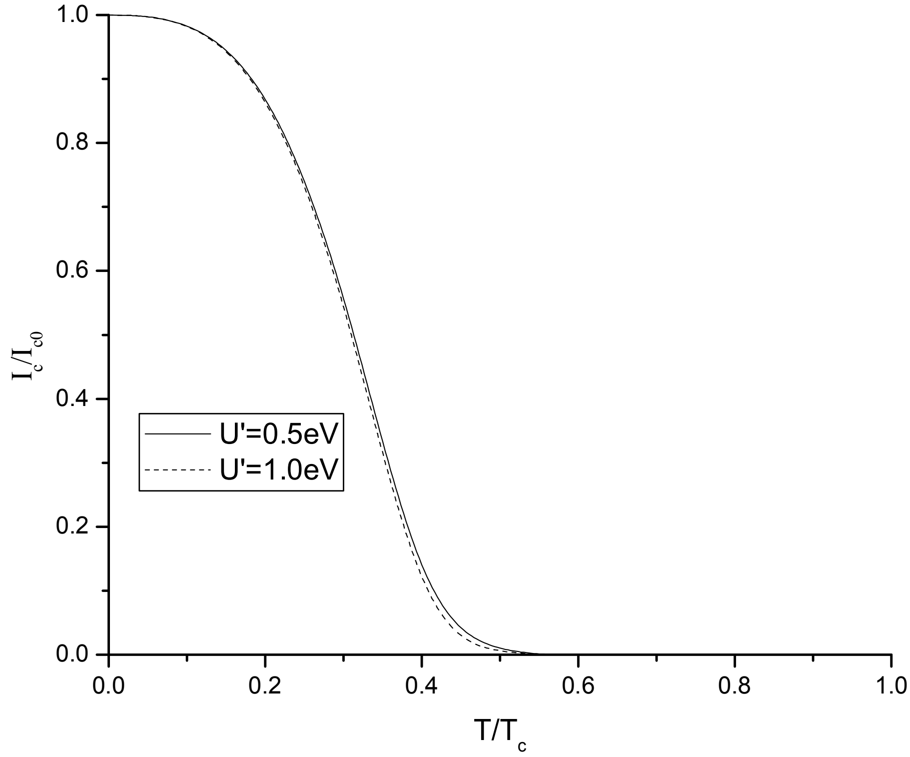

Fig. 1: Renormalized Josephson current (I_c/I_{c0}) versus T/T_c for different value of the Coulomb blockade energy (U′) keeping other parameter (V=0.01eV, ε=0.001eV, ω_c =0.025eV and n=1) fixed.

In Figure 2, we have plotted the renormalized Josephson supercurrent versus level energy on the dot (ε) for different Coulomb energy (U′) keeping other parameters (V=0.01eV, ω_c=0.025eV, T=12K and n=1) fixed. It is evident from Figure 2 that for large value of on dot Coulomb energy the renormalized Josephson supercurrent across the S-QD-S junction decreases because of blocking of charge carrier. Suppression in the Josephson supercurrent due to the strong on dot Coulomb interaction has also been predicted by various groups [24-26].

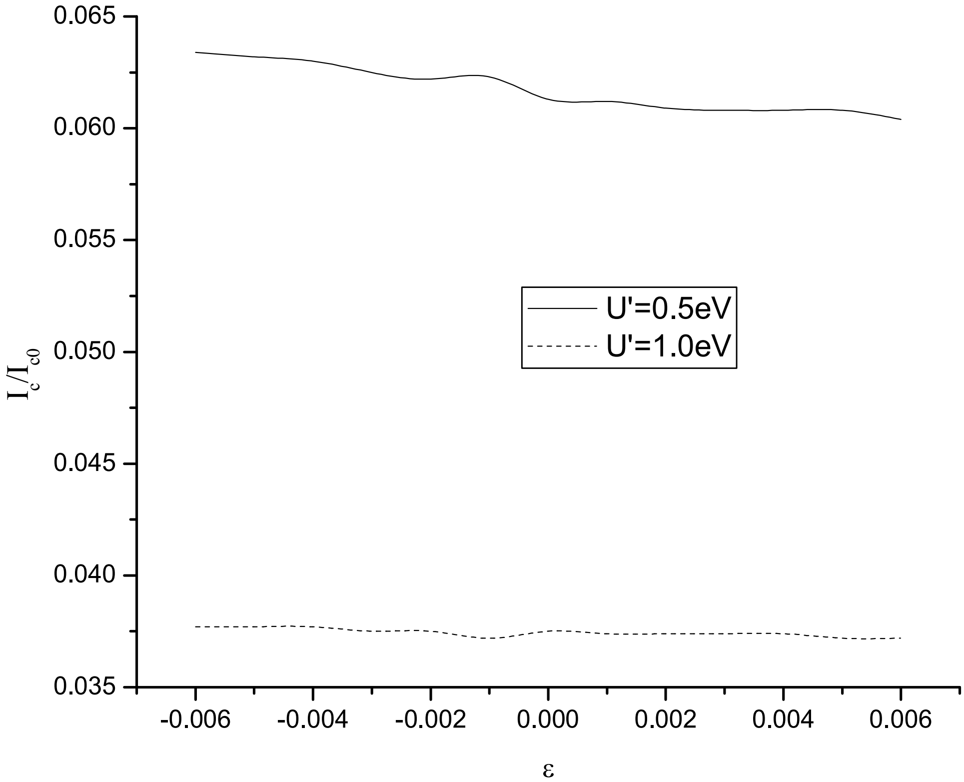

Fig. 2: Renormalized Josephson current (I_c/I_{c0}) versus dot energy level (ε) for different value of the Coulomb blockade energy (U′) keeping other parameter (V=0.01eV, T=12K, ω_c =0.025eV and n=1) fixed.

Finally, it can be concluded that on dot Coulomb correlation suppresses the Josephson supercurrent and at higher temperature (but still $T<T_c$) on dot Coulomb energy play a dominant role on Josephson supercurrent. The present calculations are based on mean-field approximation to treat strong on dot Coulomb energy. Also it will be interesting to introduce the Kondo effect parameter in the presence of finite on dot Coulomb energy in the model Hamiltonian to improve the results for better comparison with the experimental finding.

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