Study of Shock Wave in Non-Ideal Gas

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One dimensional propagating gravitating gas behind a shock wave propagating in a uniform non-ideal gas at rest is investigated. The equation of state in the suitable form for ideal gases is found to be much accurate and the equation of state for non-ideal gas is considered as given by Landau and Lifsitz.

Keywords: Shock wave, Non-ideal gas.

1. INTRODUCTION

Chisnell [1] published his dissertation on shock waves. Their equations for shock in particle velocity, stress and specific internal energy have been studied. These conditions are derived from the conservation law of mass, momentum and energy under the assumption that the shock is a single unsteady wave front with no thickness. In an ideal gas Sedov [2] have studied adiabatic flow in self-gravitating gas and obtained numerical solutions assuming that the total energy content of the flow is constant. C. Purohit *et al.* [3] has discussed the homo thermal flow of self- gravitating gas behind a shock wave on the same assumption. He has also obtained a particular analytical solution taking the temperature gradient as behind the shock wave assuming different assumptions such as energy is increasing and gas is rotating**.**

2. METHADOLOGY

In the present paper, the study of self-gravitating flow behind the spherical shock wave in non-ideal gas has been studied in which shock wave radius varies as some power of time. The medium ahead of the shock wave surface is assumed to be uniform. In this paper non ideal gas has its importance at high temperature because the validity of the assumption of the gas being ideal at low temperature.

Now we take the equation of state for non ideal gas as obtained by L.D. Londau and E.M. Lifeshitz [4], considering an expansion of the power p in powers of the density of gas.

$$P \quad = \lceil\rho \;\; T[1 + \rho c_1(T) + \rho^2 c_2(T) + ...]$$

where Γ is the gas constant P, ρ and T are the pressure, density and temperature of the non-ideal gas respectively and $c_1(T)$, $c_2(T)$ are viril coefficients.

For gases *b*ρ<< 1, *b* being the internal volume of the molecules per unit mass and therefor it is sufficient to consider the equation of state in the form [5,6,7] as

$$P=\Gamma \rho T[1 + b\rho] \tag{1}$$

The total energy of the flow between the shock wave and contact surface is constant. Here the shock wave is assumed to be strong and propagating in a uniform medium at rest.

2.1. Basic Equations

One dimensional equations of motion governing the flow of the gas, taking mass, momentum and energy are

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial r}(\rho u)+\frac{2\rho u}{r}=0 \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{Gm}{r^2}=0 \tag{3}$$

$$\frac{\partial m}{\partial r}=4\pi\rho r^2 \tag{4}$$

$$\frac{\partial e}{\partial t}+u\frac{\partial e}{\partial r}+\rho\left[\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)+\frac{u\partial}{\partial r}\left(\frac{1}{\rho}\right)\right]=0 \tag{5}$$

where the specific internal energy, $e=\dfrac{p}{(r-1)(1+b\rho)\rho}$ (6)

where 'γ' is the adiabatic index equation implies that

$$c_p - c_v=\Gamma\ (1 + b^2\rho^2/(1 + 2bq) \cong \Gamma \tag{7}$$

where Γ being gas constant. G is gravitational constant and *u*, ρ, *p*, *m*, *r*, *t* and *T* are velocity, density, pressure, mass contained is a sphere of radius r, time and temperature of the gas respectively.

2.2. Boundary Conditions

Boundary conditions for strong shock waves in present study are

$$u_1 = (1-\beta)\frac{\partial R}{\partial t} \tag{8}$$

$$\rho_1 = (1-\beta)\rho_0 \frac{\partial^2 R}{\partial t^2} \tag{9}$$

$$\rho_1 = \rho_0/\beta \tag{10}$$

$$m_1 = m_0 \tag{11}$$

where $\frac{\partial R}{\partial t}$ is shock speed, R is shock wave radius and β is obtained by the relation:

$$\beta = \frac{(r-1-\alpha)+\sqrt{(r-a-\alpha)^2+4(r+1)}}{2(r+1)} \tag{12}$$

where $$\alpha = b\rho_0(\gamma-1) \tag{13}$$

The field variables immediately ahead at the shock wavefront are

$$u = 0,\ \rho_0 = \text{constant},\ m_0 = \frac{4}{3}\pi\rho_0 R^3 \text{ and } p_0 = -\frac{2}{3}\pi G\rho_o^2 R^2 \tag{14}$$

2.3. Differential Equations

Non-dimensional variables for exact numerical solutions are

$$u = \frac{\partial R}{\partial t} f,\ \rho = \rho_0 D(x) \text{ and } p = \rho_0 \frac{\partial^2 R}{\partial t^2} p(x) \tag{15}$$

$$m = \frac{\partial^2 R}{\partial t^2} R w(x),\ \text{where } x = r/R. \tag{16}$$

and $$R = At \tag{17}$$

where f, D, p, w are functions of x and A is a constant.

The mach number is defined as

$$M^2 = \frac{\partial^2 R}{\partial t^2}\frac{\rho_0}{\gamma\rho_0} \tag{18}$$

Since the steller model of infinite radius is initially in hydrostatic equilibrium due to self gravitating, therefore

$$G = \left(-\frac{1}{\gamma m^2}\right)\left(\frac{3}{2\pi\rho_0 R^2}\frac{\partial^2 R}{\partial t^2}\right)$$

Which after modification becomes

$$G = \frac{-3\delta^2}{2\rho m^2 x^2 \frac{(\delta-1)}{\delta}} \tag{19}$$

Substituting the values from equation (15) and (16) in equations (5) and (8) the transformed basic equations can be written as

$$f' = \frac{2PNf - \frac{2D}{rm^2x^2}(x-f) + \frac{(\delta-1)}{\delta}\left[\frac{2p}{D} - f(x-f)\right]}{[(x-f)^2 - PNx]} \tag{20}$$

$$D' = \frac{D[2f + f'(x)]}{(x-f)} \tag{21}$$

$$p' = \frac{p}{(n-f)}[(2f - f'(x)DN - \frac{2(\delta-1)}{\delta}] \tag{22}$$

$$W' = 2wx^2 \tag{23}$$

$$\text{Where } N = \left[\frac{\alpha}{[(r-1)+\alpha D]} + \frac{\gamma}{D} + \alpha\right] \tag{24}$$

and the prime denotes the differentiation with respect to *x*.

The transform of the shock conditions are

$$f(1) = (1-\beta) \tag{25}$$

$$D(1)= \frac{1}{\beta} \tag{26}$$

$$P(1)= (1 - \beta) \tag{27}$$

$$W(1)= 8n/3\delta^2 \tag{28}$$

3. RESULT AND DISCUSSION

Numerical estimates of flow variables have been computed only at those location of the shock front which are permitted by the initial entropy conditions for $r = \frac{4}{3}$, δ = 0.819, M^2 =11 and α=0.025, 0.050, 0.075. From the shock front with transform boundary conditions, it is observed that the values of the density ratio *p* changes accordingly as the parameter α changes for certain values as changes the value of the flow variables. We observe that due to non- ideal nature of the gas, the nature of the flow variables is much affection. The value of velocity decreases as α increases. In the case of density, pressure and mass their values decreases as velocity increases but the nature of their flow variables in each case decreasing. The conclusion of this paper is that nature of flow variables differ in ideal and non-ideal gases as seen from the results.

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