The Study of MHD Flow of A Oldroyd B_ Liquid through a Porous Medium between Inclined Parallel Plates

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MHD flow of a dusty Oldroyd B_ liquid through a porous medium between two parallel plates inclined to the horizon has been studied. The liquid velocity, dust particle velocity and flux of flow have been obtained. Results of several authors have been deduced as particular cases of the present investigation. The physical situation of motion has been discussed.

Keywords: Oldroyd B liquid, Porous medium, Laminar flow.

1. INTRODUCTION

The study of dusty flows is useful due to applications in paper industry, petroleum industry, industrial filtration, ceramic engineering, powder metallurgy etc.

Oldroyd [1,2,3], Rivlin - Ericksen [4], Walter [5], etc. discussed on flow of viscoelastic liquids. Several authors [6-7] have studied problems on flow of dusty viscoelastic fluids with porous medium. Recently, Singh [8], Fransson and Talamelli [9], Chaudhary *et al.* [10] have studied problems on flow of viscoelastic fluids through porous media in presence of uniform magnetic field.

In this manuscript, dusty flow of a viscoelastic (oldroyd B_ liquid) liquid between two parallel inclined plates through porous medium has been studied. The liquid velocity, particle velocity and discharge of the flux of flow have been obtained and discussed.

2. FOMULATION OF THE PROBLEM

Let us consider laminar flow of a dusty, incompressible, electrically – conducting oldroyd B _ liquid through a porous medium between two parallel plates, placed at a distance h and inclined at an angle θ to the horizontal. The lower and upper plates move with velocity u_1 and u_2 and oscillates in their own plane with frequency $n_1 and\ n_2$ respectively. A constant magnetic field is applied perpendicular to the flow region. In Cartesian coordinate system, X – axis is taken along the flow of liquid in the plane of the lower plate and Y – axis perpendicular to it. Our analysis is based on the following assumptions.

(i) The dust particles are solid, non conducting and uniformly distributed in the flow region.

(ii) The temperature between the particles is uniform and the number density is constant throughout the motion.

(iii) Chemical reaction, mass transfer, interaction between the particles and radiation between the particles and liquid has not been considered.

(iv) The buoyancy force, induced magnetic field, Hall effect have been neglected.

Therefore, following Saffman [11], the governing equations of motion for the flow are:

$$(1+\lambda_1 \frac{\partial}{\partial t}) \frac{\partial u}{\partial t} = -\frac{1}{\rho}(1+\lambda_1 \frac{\partial}{\partial t})\frac{\partial p}{\partial x} + \nu(1+ \lambda_2 \frac{\partial}{\partial t}) \frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(1+\lambda_1 \frac{\partial}{\partial t})(v - u)$$

$$- g\sin\theta - \frac{\mu}{k}(1+\lambda_1 \frac{\partial}{\partial t})u - \frac{\sigma}{\rho} B_0^2 (1+\lambda_1 \frac{\partial}{\partial t})u \qquad (1)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + g\cos\theta = 0 \qquad (2)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = 0 \qquad (3)$$

And

$$m\frac{\partial v}{\partial t} = K(u - v) \qquad (4)$$

Where u is the velocity of liquid, v is the velocity of dust particles, N_0 is the number density of dust particles, m is the mass of dust particles, B_0 is the magnitude of magnetic field, K is the stokes resistance coefficient, λ_1 is the stress relaxation time and λ_2 is the rate of strain retardation time.

K is the permeability of the medium.

We express the pressure p as

$$P = \rho g \,(X\sin\theta + Y\cos\theta) - X\rho\,\phi(t) \qquad (5)$$

On substituting (5) in the equation (1), we obtain

$$(1 + A_1 \frac{\partial}{\partial t}) \frac{\partial u}{\partial t} = f(t) + \nu(1 + \lambda_2 \frac{\partial}{\partial t}) \frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(1+\lambda_1 \frac{\partial}{\partial t})(v - u) + \frac{\nu}{K}\,(1+\lambda_1 \frac{\partial}{\partial t})u \frac{\sigma}{\rho} {B_0}^2 (1+\lambda_1 \frac{\partial}{\partial t})u \qquad (6)$$

Where, $f(t) = \phi(t) + \lambda_{1\,\phi'(t)}$

We introduce the following non dimensional variables

$U^* = \frac{u}{h}$, $v^* = \frac{v}{h}$, $t^* = \frac{t}{\lambda_1}$, $Y^* = \frac{Y}{\sqrt{\nu\lambda_1}}$, $f^*(t) = \frac{\lambda_1 f(t)}{h}$ and $K^* = \frac{K}{\lambda_1 \mu}$

Using these non-dimensional quantities, the equations (4) and equations (6), after neglecting the asterisks over them, are reduced to

$$\omega\frac{\partial v}{\partial t} = u - v \tag{7}$$

And

$$(1+\frac{\partial}{\partial t})\frac{\partial u}{\partial t} = f(t) + (1 + E\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial Y^2} + \frac{l}{\omega}(1+\frac{\partial}{\partial t})(v - u) - (M^2 + \frac{1}{K})\left(1+\frac{\partial}{\partial t}\right)u \tag{8}$$

Where $l = \frac{mN_0}{\rho}$, $\omega = \frac{m}{K\lambda_1}$, $E = \frac{\lambda_2}{\lambda_1}$ (viscoelastic parameter) and $M = B_0\sqrt{\frac{\sigma\lambda_1}{\rho}}$ (Hartmann number).

The non dimensional boundary conditions are

$$u = v = u_1 e^{-n_1 t} \text{ at } Y = 0 \tag{9}$$

$$u = v = u_2 e^{-n_2 t} \text{ at } Y = \frac{h}{\sqrt{\nu\lambda_1}} = H \text{ (say)}$$

3. SOLUTION OF THE PROBLEM

Following Lihthill [12], we assume the proper solution of the equations (7) and (8) in the following form:

$$\left.\begin{aligned} u(y,t) &= u_1 f(y)e^{-n_1 t} + u_2 g(y)e^{-n_2 t} \\ v(y,t) &= u_1 F(y)e^{-n_1 t} + u_2 G(y)e^{-n_2 t} \\ and f(t) &= u_1 c_1 e^{-n_1 t} + u_2 c_2 e^{-n_2 t} \end{aligned}\right\} \tag{10}$$

Where c_1 and c_2 are arbitrary constants.

Substituting the values of u(y,t), v(y,t) and f(t) from (10) in the equations (7) and (8) and comparing coefficients of u_1 and u_2, we obtain

$$F(y) = \frac{f(y)}{1-\omega n_1} \tag{11}$$

$$G(y) = \frac{g(y)}{1-\omega n_1} \tag{12}$$

$$f''(y) + {d_1}^2 f(y) = -d_2 \tag{13}$$

$$\text{And } g''(y) + {d_3}^2 g_2(y) = -d_4 \tag{14}$$

Where, $${d_1}^2 = \left[M^2 + \frac{1}{k} - n_1 + \frac{ln_1}{1-\omega n_1}\right]\frac{(n_1-1)}{(1-n_1E)} \text{ and } d_2 = \frac{c_1}{1-n_1E}$$

$${d_3}^2 = \left[M^2 + \frac{1}{k} - n_2 + \frac{ln_2}{1-\omega n_2}\right]\frac{(n_2-1)}{(1-n_2E)} \text{ and } d_4 = \frac{c_2}{1-n_2E}$$

The transformed boundary conditions are

$f(y) = 1, g(y) = 0, F(y) = 1, G(y) = 0 \ at \ y$=0 and

$$f(y) = 0, g(y) = 1, F(y) = 0, G(y) = 1 at \ y = 0 \quad (15)$$

On solving the equations (13) and (14) under the boundary conditions (15), we get $f(y) \ and \ g(y)$, and then using (11), (12) and (10) we get the liquid velocity and particle velocity as

$$u(y,t) = \frac{u_1 e^{-n_1 t}}{{d_1}^2 sind_1 H}[d_2(1 - cosd_1 H)sind_1 y - d_1^2 cosd_1 H sind_1 y + d_1^2 sind_1 H cosd_1 y - d_2(1 - cosd_1 y)sind_1 H]$$

$$+ \frac{u_2 e^{-n_2 t}}{d_3^2 \, sind_3 H}[d_4(cosd_3 y - 1)sind_3 H + d_3^2 sind_3 y + d_4(1 - cosd_3 H)sind_3 y] \quad (16)$$

$$v(y,t) = \frac{u_1 e^{-n_1 t}}{{d_1}^2(1 - \omega n_1)sind_1 H}\begin{bmatrix} d_2(cosd_1 y - 1) - d_1^2\,(1 - \omega n_1)cosd_1 H sind_1 y \\ +d_1^2(1 - \omega n_1)sind_1 H cosd_1 y + d_2(1 - cosd_1 H)sind_1 y \end{bmatrix}$$

$$+ \frac{u_2 e^{-n_2 t}}{d_3^2\,(1-\omega n_2)sind_3 H}\begin{bmatrix} d_4(cosd_3 y - 1)sind_3 H + d_3^2(1 - \omega n_2)sind_3 y \\ +d_4(1 - cosd_3 H)sind_3 y \end{bmatrix} \quad (17)$$

3.1. Flux of Flow

The discharge of the flux per unit width of the plates for fluid and dust particles, are given by

$$\Omega_1\Omega_2 = \pi \int_0^H y u(y,t) v(y,t)\, dy \quad (18)$$

$$\Omega_1 = \frac{\pi u_1 e^{-n_1 t}}{2{d_1}^3 sind_1 H}\left[2{d_1}^2 H - (d_2 H^2 + 2d_1)sind_1 H - 2d_2 H(1 + cosd_1 H)\right]$$

$$+ \frac{\pi u_2 e^{-n_2 t}}{2{d_3}^3 sind_3 H}\left[2d_4 H(1 - cosd_3 H) - (d_4 H^2 - 2)sind_3 H - 2{d_3}^2 H cosd_3 H\right] \quad (19)$$

$$\Omega_2 = \frac{\pi u_2 e^{-n_1 t}}{2{d_1}^3(1-\omega n_1)sind_1 H}\begin{bmatrix} 2{d_1}^2 H(1 - \omega n_1) - (d_2 H^2 + 2d_1)sind_1 H \\ -2d_2 H(1 + cosd_1 H) \end{bmatrix}$$

$$+ \frac{\pi u_2 e^{-n_2 t}}{2{d_3}^3(1-\omega n_2)sind_3 H}\begin{bmatrix} 2d_4 H(1 - cosd_3 H) - (d_4 H^2 - 2)sind_3 H \\ -2{d_3}^2 H(1 - \omega n_2)\, cosd_3 H \end{bmatrix} \quad (20)$$

Particular case:

Case 1: When u_1=0 , u_2=-1, n = 0 and $\frac{1}{k}$ = 0, then the results are exactly same as obtained by Singh [8].

Case 2: When $u_1 = u_2$=1, $n_1 = 0 \; and \; \frac{1}{k} = 0,$ then the results are exactly same as obtained by Uday Raj [13].

Case 3: When $u_1 = 0, \; n_2 = 0, \; E = 0 \; and \; \frac{1}{k} = 0,$ then the results are exactly same as obtained by Chaudhary and Singh [10].

4. DISCUSSION AND CONCLUSIONS

Equations (16) and (17) give the velocity profiles of the liquid and dust particles on different magnitudes of the magnetic field, for various values of the porosity of the medium, for various values of viscoelastic parameter and for various values of Y. We concluded that:

(i) The velocity of the liquid and dust particles decreases as the intensity of the magnetic field increases.

(ii) The velocity of the liquid and dust particles increases as the porosity of the medium increases.

(iii) On increasing the viscoelastic parameter E the velocity of the liquid and dust particles increases slowly.

(iv) The velocity of the liquid and dust particles decreases as Y increases.

(v) As time ‘t’ increases the velocity of the liquid and dust particle decreases.

REFERENCES

[1] J.G. Oldroyd; “A rational formulation of the equations of plastic flow for a Bingham solid”, Mathematical Proceedings of the Cambridge Philosophical Society, Vol. 43(01), pp. 100-105, Jan 1947.

[2] J.G. Oldroyd; “Rectilinear plastic flow of a Bingham solid”, Mathematical Proceedings of the Cambridge Philosophical Society, Vol. 43(03), pp. 396 - 405, July 1947.

[3] J. G. Oldroyd; “Non-Newtonian Effects in Steady Motion of Some Idealized Elastico-Viscous Liquids”, Proceedings of the Royal Society A, Vol. 245(1241), pp. 278-297, June 1958.

[4] R.S. Rivlin and J.L. Ericksen; “Stress-deformation relations for isotropic materials”, Journal of Rational Mechanics Analysis, Vol. 4(2), pp. 323-425, 1955.

[5] K. Walters; "The motion of an elastico-viscous liquid contained between coaxial cylinders (II)", Quarterly Journal of Mechanics & Applied Mathematics, Vol. 13(4), pp. 444-461, 1960.

[6] P.N. Kaloni; "Fluctuating Flow of an Elastico-Viscous Fluid Past a Porous Flat Plate", Physics of Fluid, Vol. 10 (6), pp. 1344-1346, 1967.

[7] P. Kumar and N.P. Singh; "MHD hele-shaw flow of an elasticoviscous fluid through porous media", Bulletin of Calcutta Mathematical Society, Vol. 81, pp 32-41, 1989.

[8] A.K. Singh, "Effect of mass transfer on free convection in MHD flow of a viscous fluid", Indian Journal of Pure & Applied Physics, Vol. 41, pp 262-274, 2003.

[9] J.H.M. Fransson and Alessandro Talamelli; "On the generation of steady streamwise streaks in flat-plate boundary layers", Journal of Fluid Mechanics, Vol. 698, pp 211- 234, May 2012.

[10] R.K.S. Chaudhary and K.K. Singh; "Flow of a dusty viscoelastic (Kuvshiniski model) liquid down an inclined plane", Proceedings of the National Academy of Sciences, India, Vol. 61A , pp. 223-228, 1991.

[11] P.G. Saffman; "On the stability of laminar flow of a dusty gas", Journal of Fluid Mechanics, Vol. 13, pp. 120-128, 1962.

[12] M.J. Lighthill; "The Response of Laminar Skin Friction and Heat Transfer to Fluctuations in the Stream Velocity", Proceedings of the Royal Society A, Vol. 224(1156), pp. 1–23, 1954.

[13] Uday Raj; Ph. D. Thesis, Agra Univ. Chap. VIII, pp. 225-239, 1992.