M*- Separation Axioms

Dr. M.C. Sharma[1,*] and Poonam Sharma[2]
sharmamc2@gmail.com and poonamsachin83@gmail.com
[1,*,2] Department of Mathematics, N.R.E.C. College, Khurja – 203131 (U.P.) India

In this paper, we introduced the concepts of new separation axioms called M-separation axioms i. e. M*-$T_{½}$, M*-T_b, M*-T_d - spaces by using M* - open sets in topological space and obtained several properties of such spaces.*

Keywords: gM*- closed, gM*- open, rgM*- open sets, M*-$T_{½}$, M*-T_b, M*-T_d - spaces.

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1. INTRODUCTION

In this paper, we introduced the concepts of new separation axioms called M*-separation axioms i.e. M*-$T_{½}$, M*-T_b, M*-T_d by using M*-open sets due to Palaniappan et al. [1] in topological space and obtained several properties of such spaces. In 2012, Carpintero et al. [2] introduced a new separation axioms i.e. b- γ-T_0, b- γ-T_1, b- γ-T_2 in topological spaces by using b-open, γ-open sets and obtained several properties of such spaces.

2. PRELIMINARIES

2.1 Definition

A subset A of a topological space X is called

(i) **α*- set** [3], if int(cl(int(A))) = int(A).

(ii) **C - set** [3], if A = U $\cap$ V, where U is an open and V is an α* - set in X.

(iii) **αCg - closed** [4], if α - cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is C - set in X.

(iv) **M*-closed** [1], if α - cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is αCg - open in X.

(v) **M*g - closed** [5], if M*-cl(A) $\subset$ U whenever A $\subset$ U and U is open in X.

(vi) **gM* - closed** [5], if M*-cl(A) $\subset$ U whenever A $\subset$ U and U is M*-open in X.

The complement of αCg - closed (resp. M* - closed, M*g - closed, gM*- closed) set is said to be **αCg - open** (resp. **M* - open**, **M*g - open, gM*- open**) set. The intersection of all M*- closed subsets of X containing A is called the **M*- closure of A** and is denoted by **M*- cl(A)**. The union of all M*- open subsets of X in which are contained in A is called the **M*- interior of A** and is denoted by **M*- int(A)**. The family of M*- open (resp. M* -

closed) sets of a topological space X is denoted by **M*O(X)** (resp. **M*C(X)**).

2.2. Remarks

Remark 2.2.1. [1]

Every α-closed (resp. α-open) set is M*- closed (resp. M*-open) set.

For definitions stated above, we have the following diagram:

$$\begin{array}{ccccccc} \text{closed} & \Rightarrow & \alpha\text{ - closed} & \Rightarrow & g\alpha\text{ - closed} & \Rightarrow & \alpha g\text{ - closed} \\ & & \Downarrow & & \Downarrow & & \Downarrow \\ & & M^*\text{ - closed} & \Rightarrow & gM^*\text{ - closed} & \Rightarrow & M^*g\text{- closed.} \end{array}$$

However the converses of the above are not true as may be seen by the following examples:

Example 2.2.1: Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Then A = {c} is α- closed set as well as M*- closed set but not closed set in X.

Example 2.2.2: Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Then the set A = {c} is gM* - closed set but not closed set in X.

Remark 2.2.2.

(i) A subset A of X is M*g-open in X iff F⊂ M*-int(A) whenever F⊂U and F is closed in X.

(ii) A subset A of X is gM*-closed (resp. gM* - open) in X iff A is g-closed (resp. g-open) in X.

3. M*- T_1, M*-$T_{½}$, M*-T_b, M*-T_d - SPACES

3.1. Definition

A topological space X is said to be

(i) **T_1** (resp**. M*-T_1**), if for any distinct pair of points x and y in X, there exists an open (resp. M*-open) set U in X containing x but not y and open (resp. M*- open) set V in X containing y but not x.

(ii) A **$T_{½}$** [6], if every g-closed set is closed.

(iii) A **M*-$T_{½}$**, if every gM*-closed set is M*-closed.

(iv) A **M*-T_b**, if every M*g - closed set in X is closed.

(v) A **M*-T_d**, if every M*g - closed set in X is g - closed.

$$\begin{array}{ccccccc} T_1 & \Rightarrow & T_{½} & \Leftarrow & M^*\text{-}T_b & \Rightarrow & M^*\text{-}T_d \\ \Downarrow & & \Downarrow & & & & \\ M^*\text{-}T_1 & \Rightarrow & M^*\text{-}T_{½} & & & & \end{array}$$

It can be explained by the following example:

Example 3.1.1: A M*-T_b space need not be M*- T_1. Let X = {a, b, c} and τ = {ϕ, {a}, {b}, {a, b}, {b, c}, X}. Since {b} is not M*- closed it is not M*-T_1 and hence it is not T_1. However the family of all M*g- closed sets coincides with one of all closed sets and hence it is a M*-T_b.

3.2. Theorms

Theorem 3.2.1.

(i) X is a **$T_{½}$** iff for each x$\in$ X, {x} is open or closed in X.

(ii) X is a **M*-$T_{½}$** space iff for each x$\in$X, {x} is M*open or M*- closed in X, i.e., X is a **M*-$T_{½}$** iff a space (X, τ^{M^*}) is $T_{½}$ **-** space.

Theorem 3.2.2.

(i) If A is M*g-closed, then M*-cl(A) $-$ A does not contain non-empty closed set.

(ii) For each x $\in$X, {x} is closed or its complement X $-$ {x} is M*g-closed in X.

(iii) For each x$\in$X, {x} is M*-closed or its complement X$-${x} is gM*-closed in X.

Theorem 3.2.3

(i) Every M*-T_b space is M*-T_d and $T_{½}$.

(ii) Every T_i - space is M* -T_i, where i = 1, ½.

(iii) Every M*-T_i space is M*-$T_{½}$.

Proof:

(i) It is obtained from **Definition 3.1.** [(ii), (iv) and (v)] and **Remark 2.2.2.** (i), (ii).

(ii) Let X be a T_1 (resp. $T_{½}$) - space and let x $\in$X. Then {x} is closed (resp. open or closed by **Theorem 3.2.2.** (i). Since every open set is M*-open, {x} is M*-closed (resp. M*-closed or M*-open) in X. This implies that X is T_1 (resp. $T_{½}$ by **Theorem 3.2.2** (ii) Therefore X is M*-T_1 (resp. M*-$T_{½}$).

(iii) Let X be a M*-T_1 space. Then X is T_1. By **Theorem 5.3** of [6] X is $T_{½}$ and hence X is M*-$T_{½}$.

3.3. Proposition

(i) If X is **M*-T_b** then for each x $\in$X, {x} is M*-closed or open in X.

(ii) If X is **M*-T_d** then for each x $\in$X, {x} is M*-closed or g-open in X.

Proof:

(i) Suppose that, for an x $\in$X, {x} is not M*-closed. By **Theorem 3.2.2.** (iii) and **Remark 2.2.2** (i) and (ii), X – {x} is M*g-closed set. Therefore X – {x} closed by using assumption and hence {x} is open.

(ii) Suppose that, for a x $\in$X, {x} is not closed. By **Theorem 3.2.2 (ii)**, X – {x} is M*g-closed set. Therefore by using assumption X – {x} is g-closed and hence {x} is g-open.

4. SEPARATION AXIOMS M*-T_b, and M*-T_d of SPACES ARE PRESERVED UNDER HOMOMORPHISMS

4.1. Definition

A map f: X → Y is said to be

(i) **pre M*-closed** if for each M*-closed set of X, f (F) is M* **-** closed set in Y.

(ii) **M* - irresolute** if for each M*-closed set F of Y, f^{-1} (F) is M*-closed in X.

4.2. Theorems

Theorem 4.2.1.

(i) A map f: X → Y is pre M*-closed (resp. pre M*-open) iff its induced map f: (X, τ^{M^*}) → (Y, σ^{M^*}) is a closed (resp. open) map.

(ii) A map f: X → Y is M*-irresolute iff its induced map f: (X, τ^{M^*}) → (Y, σ^{M^*}) is continuous.

Theorem 4.2.2.

(i) A map f: X → Y is a homomorphism, then f is a M*- homomorphism.

(ii) If X is M*-T_b (resp. M*-T_d) and f: X → Y is a homomorphism, then Y is M*-T_b (resp. M*-T_d).

Proof:

(i) Since f: X → Y is a homomorphism, then f and f^{-1} are both open and M*-continuous bijection. It follows from **Theorem 4.16** of Noiri [7] that f and f^{-1} are M*-irresolute. Therefore, f is M*-irresolute and f is pre M*-closed.

(ii) Let f: X → Y is a homomorphism and let F be a M*g-closed set of Y Then by (i) f^{-1}: Y → X is a continuous and pre M*-closed bijection. Hence by **Theorem 4.2.1.** (i), f^{-1} (F) is M*g-closed in X. Since X is M*-T_b (resp. M*-T_d), f^{-1} (F) is closed (resp. g-closed) in X. Since f is closed onto (resp. closed and continuous) map, F is closed (resp. g-closed) by **Theorem 6.1** [6] **i**n Y. Hence Y is M*-T_b (resp. M*-T_d).

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