A New Method Based on the Comparison of the Connection Strings of the Kinematic Chains to detect Isomorphism

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This paper presents a new method for the detection of isomorphism in kinematic chains which plays an important role in structural synthesis of kinematic chains. The proposed method uses the new invariants called as connection string & connection value. These invariants are based on the degree of links and used as an identifier of a kinematic chain (KC). The proposed method is easy to compute, reliable and capable of detecting isomorphism in all types of KC, i.e. chains of single or multi degree of freedom with simple joints. This study will help the designer to select the best KC and mechanisms to perform the specified task at conceptual stage of design.

Keywords: Isomorphism, Kinematic chain (KC), Connection string, Connection value, Degree of freedom

1. INTRODUCTION

One of the most difficult task in structural synthesis of kinematic chain is to check the isomorphism among two chains. The two kinematic chains KC1 and KC2 are said to be isomorphic, if there exists a one to one correspondence between the links of KC1 and KC2.

The detection of isomorphism among two kinematic chains with same link assortment is necessary to prevent duplication and omission of a potential useful chain. A lot of time and effort had been devoted in developing a reliable and computationally efficient technique, therefore a wealth of literature pertaining with this topic is available. However, there is a scope for an efficient, simple and reliable method and this paper is an attempt in this direction. Heuristic and visual methods [1] were only suitable for kinematic chains with a small number of links. Characteristic polynomial method [2] has the disadvantage of dealing with cumbersome calculations and later counter examples were also reported [3]. Quist and Soni presented a method of loops of a chain [4]. A unique index for isomorphism i.e. characteristic polynomial of structural matrix was proposed by Yan and Hwang [5], however this method is computationally uneconomical. Mruthyunjaya proposed the computerized method of structural synthesis which works on binary coding of chains [6]. Agarwal and Rao proposed Variable permanent function to identify multi loop kinematic chains [7]. Ambekar and Agarwal presents a method of Canonical coding of kinematic chains [8] but it becomes computationally uneconomical when applied to large kinematic chain. Hamming number technique [9] is very reliable and computationally efficient, however when the primary Hamming string fails, the time consuming computation of the secondary Hamming string is needed. Shin and Krishna

Murthy presents some rules for relabeling its vertices canonically for a given kinematic chain [10]. However it tends to become computationally inefficient where a higher number of symmetry group elements in the kinematic chain are present. The degree code [11] of the contracted link adjacency matrix of a chain was also proposed to test the isomorphism. Yadav and Pratap present a method of link distance for the detection of isomorphism [12]. A method based on artificial neural network theory by Kong et al. was presented [13]. A method based on loop formations of a kinematic chain was proposed by Rao and Prasad [14]. A new method based on eigenvalues and eigenvectors of adjacent matrices of chains was also proposed [15]. The reliability of the existing spectral techniques for isomorphism detection was challenged by Sunkari, R.P., and Schmidt [16]. Huafeng Ding and Zhen Huang [17] shows that the characteristic polynomial and eigen value approach fails and proposed a method based on the perimeter topological graph and some rules for relabeling its vertices canonically and one-to-one descriptive method for the canonical adjacency matrix set of kinematic chains Hasan and Khan [18] presented a method based on degrees of freedom of kinematic pairs. All the above methods developed so far only uses the graphs of the KC or their adjacency matrices in one or the other way. The method presented in this paper uses ‘connection string’ to detect isomorphism in KCs.

2. **PRELIMINARIES**

The following definitions are to be understood clearly before applying this method.

(i) **Degree of link:** Degree of a particular link depends upon its type the degree of a binary link is 2, the degree of a ternary link is 3, the degree of a quaternary link is 4 and so on.

(ii) **Connection value:** Connection value is ratio of the sum of the degrees of the other links connected to the particular link under consideration to degree of the same link.

(iii) **Connection string:** connection string for a chain is formed by arranging connection valve in increasing order as [a(x)……..b(y)] where a, b are the connection values and x, y are the link numbers and if more than one link has same connection values they appears only once in the string as c (i, j, k)

3. IDENTIFICATION OF ISOMORPHISM

If the connection string of the two kinematic chains has one to one correspondence then the two chains are isomorphic. Although no mathematical proof of this method is being offered but no counter example has been found by the author among the known cases of planar kinematic chains.

3.1 Basis of the Test

The kinematic chains are composed of binary, ternary, and the other higher order links. These links are joined together by simple or multiple joints. Before comparing the two chains it is necessary to consider type of links and connection of the links with the other links in the assembly. According to graph theory, the necessary condition for isomorphism is that the number and degree of vertices must be same, which means that the number and type of links of must be same in the chains to be checked for isomorphism. The type of links can be identified by their degrees. Thus a binary link has a value two, ternary three, quaternary four and so on. Degrees of links are used to determine connection values and later connection values are used form connection strings to check isomorphism.

4. APPLICATION TO KINEMATIC CHAINS

The proposed method for isomorphism detection in this paper is tested on several kinematic chains available in the literature by forming their connection strings and found that it works well in all cases. Three examples of ten links kinematic chains are shown below to reveal the reliability of the method of comparison of connection strings. We used formula as

$$\text{Connection value} = \frac{\text{sum of the degrees of the links connected to the } i^{th} \text{ link}}{\text{degree of the } i^{th} \text{ link}}$$

Example 1:- The two Kinematic chains with 10 links, single-degree of freedom shown in Figure 1(a) and (b). The aim is to examine whether these two KC are isomorphic as reported in the literature [13].

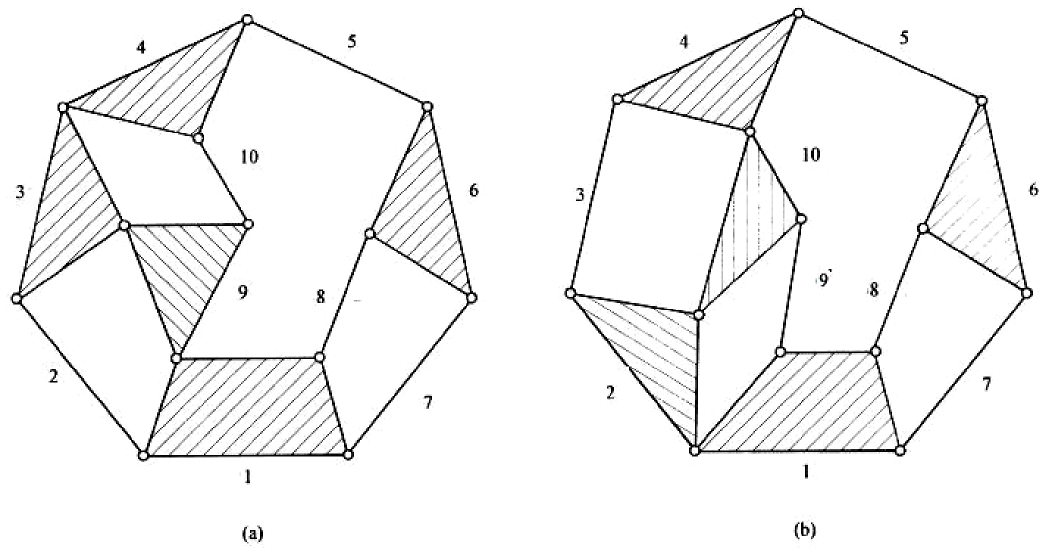

Fig. 1(a) and 1(b): The two ten link chains with single degree of freedom.

Connection values of chain for the chain shown in Figure 1(a) are {2.25, 3.5, 2.66, 2.33, 3, 2, 3.5, 3.5, 3, 3}. Hence connection string is [2(6)-2.25(1)-2.33(4)-2.66(3)-3(5,9,10)-3.5(2,6,7)]. Similarly connection values of chain for the chain shown in Figure 1(b) are {2.25, 3, 3, 2.33, 3, 2, 3.5, 3.5, 3.5, 2.66}. Hence connection string is [2(6)-2.25(1)-2.33(4)-2.66(10)-3(2,3,4)-3.5(7,8,9)].

By comparing the connection strings of two chains as shown in Figures 1(a) and 1(b), it is found that the connection string of the two chains are same hence there is one to one correspondence between the links of chains hence the two chains are isomorphic.

Example 2:- The two Kinematic chains with 10 links, single degree of freedom having same characteristic polynomials for their adjacency matrices are shown in Figures 2(a) and 2(b) The aim is to examine whether these two KC are non-isomorphic as reported in the literature [3].

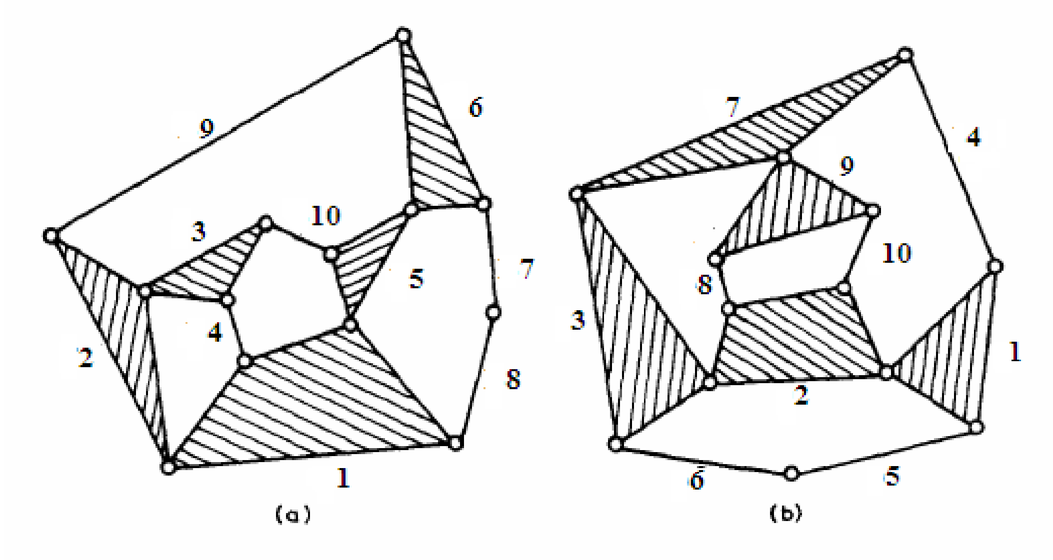

Fig. 2(a) and 2(b): The two ten link chains with single degree of freedom having same characteristic polynomial for their adjacency matrix.

Connection values of chain for the chain shown in Figure 2(a) are {2.5, 3, 2.33, 3.5, 3, 2.33, 2.5, 3, 3, 3}. Hence connection string is [2.33(2,6)-2.5(1,7)-3(2,5,8,9,10)-3.5(4)]. Similarly connection values of chain for the chain shown in Figure 2(b) are {2.66, 2.5, 3, 3, 2.5, 2.5, 2.66, 3.5, 2.33, 3.5}. Hence connection string is [2.33(9)-2.5(2,5,6)-2.66(1,7)-3(3,4)-3.5(8,10)].

By comparing the connection strings of two chains shown in Figures 2(a) and 2(b), it is found that the connection string of the two chains are not same and hence there is no one to one correspondence between the links of chains hence the two chains are non-isomorphic.

Example 3:- The two Kinematic chains with two 10 link three degree of freedom, having same characteristic polynomials for their adjacency matrices are shown in Figures 3(a) and 3(b). The aim is to examine whether these two KC are non-isomorphic as reported in the literature [3].

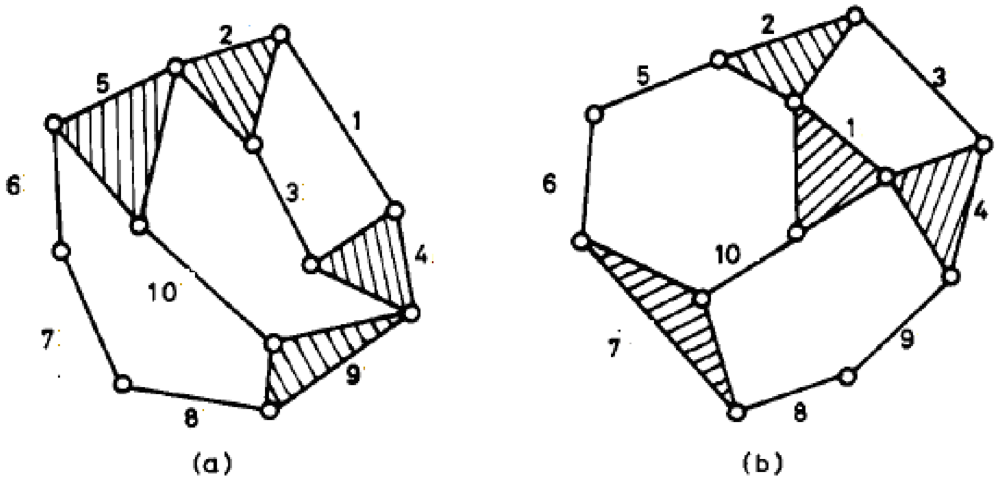

Fig. 3(a) and (b): The two ten link chains with three degree of freedom having same characteristic polynomial for their adjacency matrix.

Connection values of chain for the chain shown in Figure 3(a) are {3, 2.33, 3, 2.33, 2.33, 2.5, 2, 2.5, 2.33, 3}. Hence connection string is [2(7)-2.33(2,4,5,9)-2.5(6,8)-3(1,3,10)]. Similarly connection values of chain for the chain shown in Figure 3(b) are {2.66, 2.5, 3, 2.33, 2.5, 2.5, 2, 2.5, 2.5, 3}. Hence connection string is [2(7)-2.33(4)-2.5(2,5,6,8,9)-3(3,10)].

By comparing the connection strings of two chains shown in Figures 3(a) and 3(b), it is found that the connection string of the two chains are not same and there is no one to one correspondence between the links of chains hence the two chains are non-isomorphic

5. RESULTS

The proposed method can detect isomorphism among all kinematic chains of single or multi degree of freedom having simple joints, up to 10-links. All 16 eight-bar single-DOF kinematic chains, 40 nine-bar two-DOF kinematic chains, 98 ten-bar three DOF kinematic chains and 230 ten-bar single-DOF kinematic chains have been tested with this method and no counterexamples have been found.

6. CONCLUSION

The reliability of this method is based on the fact it has been found to be successful in not only distinguishing all known 16 kinematic chains of 8-links, 230 kinematic chains of 10-links having 1-degree of freedom and 40 kinematic chains of 9-links 2-degree of freedom but also on all the counter examples reported earlier literature. The method is so simple that one can detect isomorphism without computer program, where as most of the methods developed so far requires sophisticated algorithms.

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