Weakly g-Continuous Mappings

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The present paper comprises some basic properties of weakly g-continuous mappings. Several necessary and sufficient conditions for weakly g-continuous mappings have been studied in section 2. A few results on composition maps are also established in the last.

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1. INTRODUCTION AND PRELIMINARIES

Throughout, this paper, (X, τ) denotes a topological space with a topology τ on which no separation axioms are assumed unless explicitly stated and for a subset A of X, cl(A) and int(A) denote the closure of A and the interior of A with respect to (X, τ) respectively. P(X) be the power set of X. Before entering into our work we recall the following.

Levine [1] introduced the notion of g-closed sets. A set A in a topological space (X, τ) is said to be generalized closed (or g-closed) set if cl(A)⊂B whenever A⊂B and B is open. A set A in (X, τ) is called g-open set if its complement X−A is g-closed. By means of these g-open sets, Levine [1] defined g-continuous mappings which carry closed (open) sets inversely onto g-closed (g-open) sets, Also the authors [2] used these g-open sets to defined weakly g-continuous mappings. A point x∈X is called g-cluster point of a subset A of X if for every g-open set B of X containing x, A∩B≠ϕ. The set of all g-cluster points of A is called the g-closure of A and is denoted by g-cl(A). Obviously a set A is g-closed if and only if g-cl(A)=A. A point x∈X is said to be a g-interior point of a subset A of X if there exists a g-open set B containing x such that B⊂A. The set of all g-interior points of A is said to be g-interior of A and is denoted by g-int(A). A set A is g-open if and only if A= g-int(A). A set N⊂X is called g-neighbourhood of x∈X, if there exists a g-open set G in X such that x∈G⊂N.

A subset A of X is said to be, preopen [1] if A⊂int(cl(A)), semi open [3] if A⊂cl(int(A)) and regular open [4] (resp. regular closed [4]) if A=int(cl(A)) (resp. A=cl(int(A))).

For a subset A of X, the following are mentioned [1]:

(a) A is g-closed (g-open) if and only if A=g-cl(A) (A=g-int(A)).
(b) g-cl(X−A)=X−g-int(A).
(c) g-cl(A) is g-closed in X.
(d) g-int(A) is g-open in X.

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2. DEFINITIONS AND CHARACTERIZATIONS

E. Ekici et al. [2] have introduced and studied the notion of weakly g-continuous mappings, which is of-course a weaker form of g-continuous mappings.

Definition 2.1:- A mapping $f : X \longrightarrow Y$ is said to be weakly g-continuous if for each x in X and each open set V of Y containing f(x), there exists a g-open set U containing x such that $f(U)\subset cl(V)$ [2].

Theorem 2.2:- For a mapping $f : X \longrightarrow Y$ the following are equivalent ::

(a) f is weakly g-continuous at $x\in X$.
(b) $x\in g\text{-}int(f^{-1}(cl(U)))$ for each neighbourhood U of f(x).

Proof:- (1)⇒(2) Let U be any neighbourhood of f(x). Then there exists a g-open set G containing x such that $f(G)\subset cl(U)$. Since $G\subset f^{-1}(cl(U))$ and G is a g-open set, then $x\in G\subset g\text{-}int(G)\subset g\text{-}int(f^{-1}(cl(U)))$. Thus $x\in g\text{-}int(f^{-1}(cl(U)))$.

(2)⇒(1) Let $x\in g\text{-}int(f^{-1}(cl(U)))$ for each neighbourhood U of f(x). Take $V= g\text{-}int(f^{-1}(cl(U)))$. This implies that $f(V)\subset cl(U)$ and V is a g-open set. Hence f is weakly g-continuous at $x\in X$.

Theorem 2.3:- For a function $f : X \longrightarrow Y$, the following are equivalent :

(1) f is weakly g-continuous.
(2) $g\text{-}cl(f^{-1}(int(cl(V))))\subset f^{-1}(cl(V))$ for every subset V in Y.
(3) $g\text{-}cl(f^{-1}(int(F)))\subset f^{-1}(F)$ for every regular closed subset F in Y.
(4) $g\text{-}cl(f^{-1}(U))\subset f^{-1}(cl(U))$ for every open subset U in Y.
(5) $f^{-1}(U)\subset g\text{-}int(f^{-1}(cl(U)))$ for every open subset U in Y.
(6) $g\text{-}cl(f^{-1}(U))\subset f^{-1}(cl(U))$ for every preopen subset U in Y.
(7) $f^{-1}(U)\subset g\text{-}int(f^{-1}(cl(U)))$ for every preopen subset U in Y.

Proof:- (1)⇒(2) Let V be any subset in Y and $x\in X-f^{-1}(cl(V))$. Then $f(x)\in Y-cl(V)$ and there exists an open set U containing f(x) such that $U\cap V=\phi$. We have $cl(U)\cap int(cl(V))=\phi$. Since f is weakly g-continuous, therefore, there exists a g-open set W containing x such that $f(W)\subset cl(U)$. Then $W\cap f^{-1}(int(cl(V)))=\phi$ and $x\in X-g\text{-}cl(f^{-1}(int(cl(V))))$. Hence, $g\text{-}cl(f^{-1}(int(cl(V))))\subset f^{-1}(cl(V))$.

(2)⇒(3) Let F be any regular closed set in Y. Then $g\text{-}cl(f^{-1}(int(F)))=g\text{-}cl(f^{-1}(int(cl(int(F)))))\subset f^{-1}(cl(int(F)))=f^{-1}(F)$.

(3)⇒(4) Let U be any open subset of Y. Since, cl(U) is regular closed in Y. Then $g\text{-}cl(f^{-1}(U))\subset g\text{-}cl(f^{-1}(int(cl(U))))\subset f^{-1}(cl(U))$.

(4)⇒(5) Let U be any open set of Y. Since $Y-cl(U)$ is open in Y, then $X-g\text{-}int(f^{-1}(cl(U))) = g\text{-}cl(f^{-1}(Y-cl(U)))\subset f^{-1}(cl(Y-cl(U)))\subset X-f^{-1}(U)$. Hence, $f^{-1}(U)\subset g\text{-}int(f^{-1}(cl(U)))$.

(5)⇒(1) Let $x\in X$ and U be an open subset of Y containing f(x). Then $x\in f^{-1}(U) \subset g\text{-}int(f^{-1}(cl(U)))$. Take $W=g\text{-}int(f^{-1}(cl(U)))$. Then $f(W)\subset cl(U)$ and hence f is weakly g-continuous at x in X.

(1)⇒(6) Let U be any preopen set of Y and $x \in X - f^{-1}(cl(U))$. There exists an open set G containing f(x) such that $G \cap U = \phi$. We have $cl(G \cap U) = \phi$. Since U is preopen,then $U \cap cl(G) \subset int(cl(U)) \cap cl(G) \subset cl(int(cl(U))) \cap G \subset cl(int(cl(U \cap G))) cl(int(cl(U \cap G))) \subset cl(U \cap G) = \phi$. Since f is weakly g-continuous and G is an open set containing f(x), there exists a g-open set W in X containing x such that $f(W) \subset cl(G)$. Then $f(W) \cap U = \phi$ and $W \cap f^{-1}(U) = \phi$. This implies that $x \in X - g\text{-}cl(f^{-1}(U))$ and then $g\text{-}cl(f^{-1}(U)) \subset f^{-1}(cl(U))$.

(6)⇒(7) Let U be any preopen set in Y. Since $Y - cl(U)$ is open in Y, then $X - g\text{-}int(f^{-1}(cl(U))) = g\text{-}cl(f^{-1}(Y - cl(U))) \subset f^{-1}(cl(Y - cl(U))) \subset X - f^{-1}(U)$. This shows that $f^{-1}(U) \subset g\text{-}int(f^{-1}(cl(U)))$.

(7)⇒(1) Let $x \in X$ and U be any open set in Y containing f(x). We have $x \in f^{-1}(U) \subset g\text{-}int(f^{-1}(cl(U)))$. Take $W = g\text{-}int(f^{-1}(cl(U)))$. Then $f(W) \subset cl(U)$ and hence f is weakly g-continuous at x in X.

Theorem 2.4:- For a mapping $f : X \longrightarrow Y$, the following are equivalent :

(1) f is weakly g-continuous.
(2) $f(g\text{-}cl(V)) \subset \theta\text{-}cl(f(V))$ for each subset V in X.
(3) $g\text{-}cl(f^{-1}(G)) \subset f^{-1}(\theta\text{-}cl(G))$ for each subset G in Y.
(4) $g\text{-}cl(f^{-1}(int(\theta\text{-}cl(G)))) \subset f^{-1}(\theta\text{-}cl(G))$ for each subset G in Y.

Proof:- (1)⇒(2) Let V be any subset in X, $x \in g\text{-}cl(V)$ and U be any open set in Y containing f(x). Then, there exists a g-open set W containing x such that $f(W) \subset cl(U)$. Since $x \in g\text{-}cl(V)$, therefore, $W \cap V \neq \phi$. This implies that $\phi \neq f(W) \cap f(V) \subset cl(U) \cap f(V)$ and $f(x) \in \theta\text{-}cl(f(V))$. Hence, $f(g\text{-}cl(V)) \subset \theta\text{-}cl(f(V))$.

(2)⇒(3) Let G be any subset of Y. Then $f(g\text{-}cl(f^{-1}(G))) \subset \theta\text{-}cl(G)$ and hence $g\text{-}cl(f^{-1}(G)) \subset f^{-1}(\theta\text{-}cl(G))$.

(3)⇒(4) Let $G \subset Y$. Since $\theta\text{-}cl(G)$ is closed in Y then $g\text{-}cl(f^{-1}(int(\theta\text{-}cl(G)))) \subset f^{-1}(\theta\text{-}cl(int(\theta\text{-}cl(G)))) = f^{-1}(cl(int(\theta\text{-}cl(G)))) \subset f^{-1}(\theta\text{-}cl(G))$.

(4)⇒(1) Let U be any open set in Y. We have $U \subset int(cl(U)) = int(\theta\text{-}cl(U))$. Thus, $g\text{-}cl(f^{-1}(U)) \subset g\text{-}cl(f^{-1}(int(\theta\text{-}cl(U)))) \subset f^{-1}(\theta\text{-}cl(U)) = f^{-1}(cl(U))$. This implies that f is weakly g-continuous mapping .

3. SOME BASIC PROPERTIES

Theorem 3.1:- If $f : X \longrightarrow Y$ is a weakly g-continuous mapping and Y is Hausdorff, then f has g-closed point inverses.

Proof:- Let $y \in Y$ and $x \in A = \{x : x \in X \text{ and } f(x) \neq y\}$. Since $f(x) \neq y$ and Y is Hausdorff, then there exists disjoint open sets G and H such that $f(x) \in G$ and $y \in H$. Since $G \cap H = \phi$, therefore $cl(G) \cap H = \phi$. We have $y \notin cl(G)$. Since f is weakly g-continuous, there exists a g-open set U containing x such that $f(U) \subset cl(G)$. Assume that U is not contained in A, then there exists a point $u \in U$ such that $f(u) = y$. Since $f(U) \subset cl(G)$, we have $y = f(u) \in cl(G)$. This is a contradiction which contradicts $y \notin cl(G)$. Hence $U \subset A$ and U is g-open set in X. This shows that A is g-open in X, equivalently, $f^{-1}(y) = \{x : x \in X, f(x) = y\}$ is g-closed in X.

Let us recall that a point $x \in X$ is said to be in the θ-closure [5] of a subset A of X, denoted by θ-cl(A) if $cl(G) \cap A \neq \phi$ for each open set G of X containing x. A is called θ-closed if θ-cl(A)=A. The complement of a θ-closed set is called θ-open set.

Theorem 3.2:- If $f^{-1}(\theta\text{-cl}(V))$ is g-closed in X for every subset V in Y, then f is weakly g-continuous.

Proof:- Let V be any subset of Y. Since, $f^{-1}(\theta\text{-cl}(V))$ is a g-closed subset in X, therefore, $g\text{-cl}(f^{-1}(V)) \subset g\text{-cl}(f^{-1}(\theta\text{-cl}(V))) = f^{-1}(\theta\text{-cl}(V))$. Thus, we have $g\text{-cl}(f^{-1}(V)) \subset f^{-1}(\theta\text{-cl}(V))$. This implies from above Theorem 3.1, that f weakly g-continuous.

Theorem 3.3:- Let $f : X \to Y$ be a mapping. If f is weakly g-continuous, then $f^{-1}(V)$ is g-closed in X for every θ-closed subset V in Y.

Proof:- Evidently $f^{-1}(V)$ is g-closed set for all θ-closed sets V in Y. Since f is weakly g-continuous, so, $g\text{-cl}(f^{-1}(V)) \subset f^{-1}(\theta\text{-cl}(V)) = f^{-1}(V)$ for a θ-closed set V in Y. This implies $g\text{-cl}(f^{-1}(V)) \subset f^{-1}(V)$. Thus, we see that $f^{-1}(V)$ is g-closed whenever V is θ-closed.

Corollary 3.4:- If $f : X \to Y$ is a weakly g-continuous then $f^{-1}(V)$ is g-open set in X for every θ-open set V in Y.

Theorem 3.5:- If $f : X \to Y$ is weakly g-continuous and $g : Y \to Z$ is continuous, then the composition $gof : X \to Z$ is weakly g-continuous.

Proof:- Let x be in X and A be an open set of Z containing g(f(x)). We have $g^{-1}(A)$ is an open set of Y containing f(x). Then there exists a g-open set B containing x such that $f(B) \subset cl(g^{-1}(A))$. Since g is continuous, then $gof(B) \subset g(cl(g^{-1}(A))) \subset cl(A)$. Thus gof is weakly g-continuous.

Theorem 3.6:- Let $f, g : X \to Y$ be weakly g-continuous mapping and Y be Urysohn. If GO(X), the collection of g-open sets in X, is closed under the finite intersection, then the set $\{x \in X : f(x)=g(x)\}$ is g-closed in X.

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