Performance Study of Single Compressor-Multi Evaporator Type Refrigeration Plant

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The author, in this paper, has considered a refrigeration plant for analysis of its some important performance measures. This refrigeration plant has a single compressor with multi-evaporators. Boolean function technique has been used to formulate and solve the mathematical model. Reliability of the system has been obtained in two cases: (i) when failure rates follow exponential time distribution, and (ii) when failure rates follow Weibull time distribution. Mean time to failure of the system has also been given at the end. One numerical example together with the graphical illustration has been appended at the end to highlight the important results of the study.

Keywords: Refrigeration plant, Multi-evaporators, Boolean function, Exponential time distribution, Weibull time distribution.

1. INTRODUCTION

These single compressor type refrigeration plants can be categorized into following:

(a) Multi-evaporator type at multi-temperature.
(b) Multi-evaporator type at same-temperature.

The functioning of evaporator together with expansion valve, is to generate the constant temperature corresponding to required state. Therefore, the different evaporators can be fixed either for same temperature or for different temperatures.

In this present study, the author's investigations are based on category (b) "Multi-evaporator type at same temperature". System configuration has been shown in Figure 1. The author's idea is to take one standby condenser [1] to improve the capability of the system. This standby condenser followed online through an imperfect switching device [2], on failure of main unit. There are two expansion values and are working in parallel redundancy [3].

2. ASSUMPTIONS

The following assumptions have been associated with this chapter:

(i) The state of every component and of the whole refrigeration plant is either good or bad.
(ii) There is no repair facility to failed unit [4].
(iii) The reliability of every component is known in advance.
(iv) The states of all components are statistically independent.

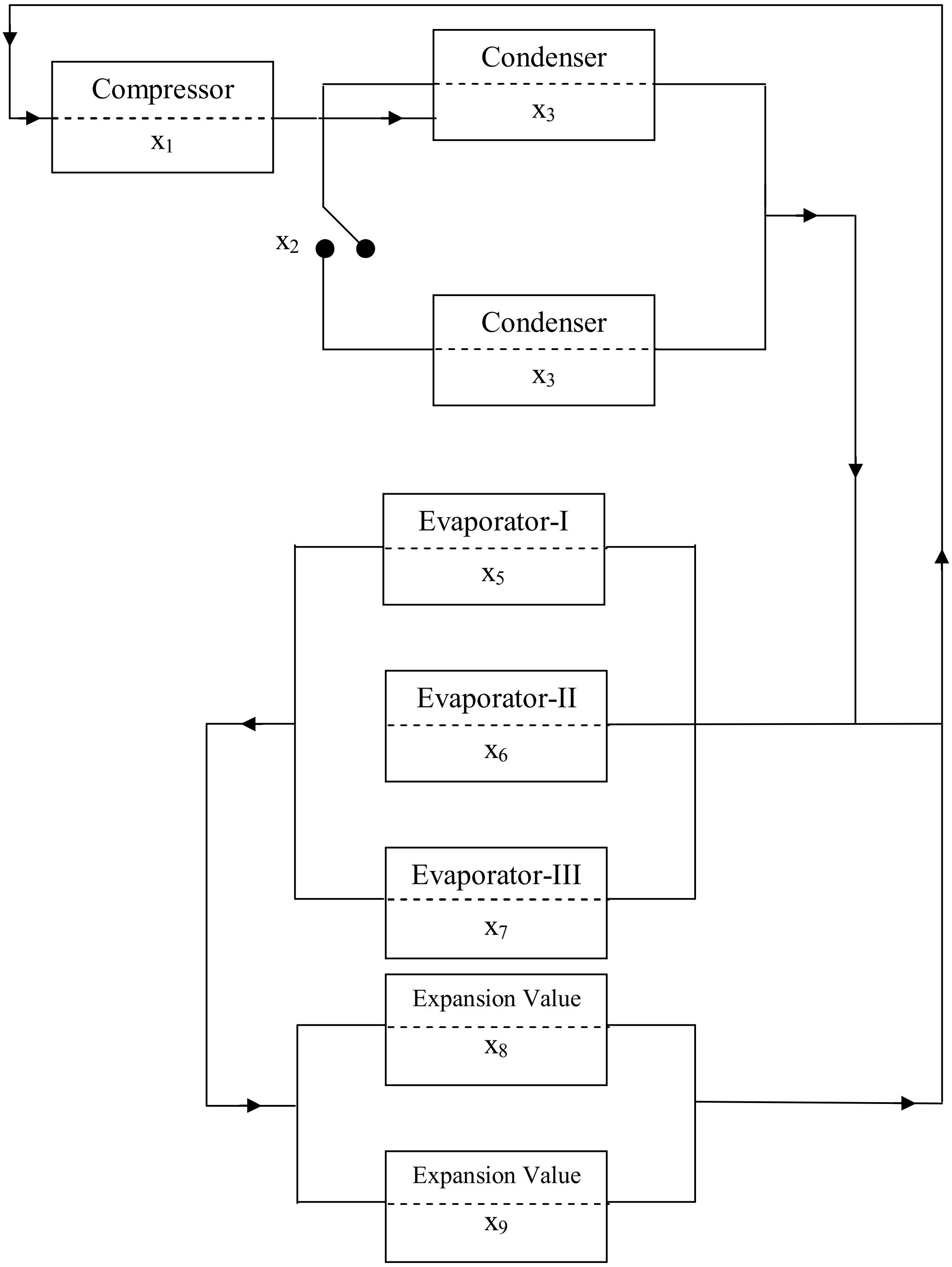

Figure 1: System Configuration

(v) The failure times of all components are arbitrary.

(vi) The three evaporators are connected in parallel and contribute same temperature. This part is of 1 – out – of 3; G nature.

(vii) There are two identical condensers connected in standby redundancy. The standby unit followed on line, through an imperfect switching device, after failure of main unit.

3. NOTATIONS

The following notations have been used throughout this study:

x_1/x_2	:	State of compressor/ switching device.
x_3, x_4	:	States of condensers.
x_5, x_6, x_7	:	States of evaporators.
x_8, x_9	:	States of expansion values.
x_i	:	Negation of x_i, for all i= 1, 2 ………9.
x_i (i = 1, 2…….9)	:	0, in bad state 1, in good state
$\wedge$, $\vee$	:	Conjunction, Disjunction

4. FORMULATION OF MATHEMATICAL MODEL

The object of the system is to contribute the same temperature generated by refrigeration plant with the help of three different evaporators [5]. Using Boolean function technique [6,7,8], the conditions of capability of the successful operation of the complex system in terms of logical matrix [8] are expressed as follows:

$$F(x_{1,}\, x_{2 \text{-------}}\, x_9) = \begin{bmatrix} x_1 & x_3 & x_5 & x_8 & \\ x_1 & x_3 & x_6 & x_8 & \\ x_1 & x_3 & x_7 & x_8 & \\ x_1 & x_3 & x_5 & x_9 & \\ x_1 & x_3 & x_6 & x_9 & \\ x_1 & x_3 & x_7 & x_9 & \\ x_1 & x_2 & x_4 & x_5 & x_8 \\ x_1 & x_2 & x_4 & x_6 & x_8 \\ x_1 & x_2 & x_4 & x_7 & x_8 \\ x_1 & x_2 & x_4 & x_5 & x_9 \\ x_1 & x_2 & x_4 & x_6 & x_9 \\ x_1 & x_2 & x_4 & x_7 & x_9 \end{bmatrix} \tag{1}$$

5. SOLUTION OF THE MODEL

By the application of algebra of logics, equation (1) may be written as:

$$F(x_1, x_2 \text{---} x_9) = [\, x_1 \wedge f(x_2, x_3 \text{---------} x_9) \,] \tag{2}$$

where,

$$f(x_2, x_3 \text{---------} x_9) = \begin{bmatrix} x_3 & x_5 & x_8 & \\ x_3 & x_6 & x_8 & \\ x_3 & x_7 & x_8 & \\ x_3 & x_5 & x_9 & \\ x_3 & x_6 & x_9 & \\ x_3 & x_7 & x_9 & \\ x_2 & x_4 & x_5 & x_8 \\ x_2 & x_4 & x_6 & x_8 \\ x_2 & x_4 & x_7 & x_8 \\ x_2 & x_4 & x_5 & x_9 \\ x_2 & x_4 & x_6 & x_9 \\ x_2 & x_4 & x_7 & x_9 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \\ T_{10} \\ T_{11} \\ T_{12} \end{bmatrix} \tag{3}$$

where,

$$T_1 = [\, x_3 \quad x_5 \quad x_8 \,] \tag{4}$$

$$T_2 = [\, x_3 \quad x_6 \quad x_8 \,] \tag{5}$$

$$T3 = [\, x_3 \quad x_7 \quad x_8 \,] \tag{6}$$

$$T_4 = [\, x_3 \quad x_5 \quad x_9 \,] \tag{7}$$

$$T_5 = [\, x_3 \quad x_6 \quad x_9 \,] \tag{8}$$

$$T_6 = [\, x_3 \quad x_7 \quad x_9 \,] \tag{9}$$

$$T_7 = [\, x_2 \quad x_4 \quad x_5 \quad x_8 \,] \tag{10}$$

$$T_8 = [\, x_2 \quad x_4 \quad x_6 \quad x_8 \,] \tag{11}$$

$$T_9 = [\, x_2 \quad x_4 \quad x_7 \quad x_8 \,] \tag{12}$$

$$T_{10} = [\, x_2 \quad x_4 \quad x_5 \quad x_9 \,] \tag{13}$$

$$T_{11} = [\, x_2 \quad x_4 \quad x_6 \quad x_9 \,] \tag{14}$$

$T_{12} = [\ x_2 \quad x_4 \quad x_7 \quad x_9\ ]$ (15)

Using orthogonalisation algorithm, equation (3) may be written as:

$$f(x_2, x_3 --, x_9) = \begin{bmatrix} T_1 \\ T_1' & T_2 \\ T_1' & T_2' & T_3 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6' & T_7 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6' & T_7' & T_8 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6' & T_7' & T_8' & T_9 \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6' & T_7' & T_8' & T_9' & T_{10} \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6' & T_7' & T_8' & T_9' & T_{10}' & T_{11} \\ T_1' & \mathrm{T}_2' & \mathrm{T}_3' & T_4' & T_5' & T_6' & T_7' & T_8' & T_9' & T_{10}' & T_{11}' & T_{12} \end{bmatrix} \quad (16)$$

Now we have:

$$T_1' = \begin{bmatrix} x_3' \\ x_3 & x_5' \\ x_3 & x_5 & x_8' \end{bmatrix}$$

$$\therefore T_1' T_2 = \begin{bmatrix} x_3' \\ x_3 & x_5' \\ x_3 & x_5 & x_8' \end{bmatrix} \wedge [x_3 \quad x_6 \quad x_8]$$

$$= [x_3 \quad x_5' \quad x_6 \quad x_8] \quad (17)$$

Similarly,

$$T_1' \mathrm{T}_2' \mathrm{T}_3 = [x_3 \quad x_5' \quad x_6' \quad x_7 \quad x_8] \quad (18)$$

$$T_1' \mathrm{T}_2' T_3' \mathrm{T}_4 = [x_3 \quad x_5 \quad x_8' \quad x_9] \quad (19)$$

$$T_1' \mathrm{T}_2' T_3' \mathrm{T}_4' T_5 = 0 \quad (20)$$

$$T_1' \mathrm{T}_2' T_3' \mathrm{T}_4' T_5' T_6 = 0 \quad (21)$$

$$T_1' \mathrm{T}_2' T_3' \mathrm{T}_4' T_5' T_6' T_7 = [x_2 \quad x_3' \quad x_4 \quad x_5 \quad x_8] \quad (22)$$

$$T_1' \mathrm{T}_2' T_3' \mathrm{T}_4' T_5' T_6' T_7' T_8 = [x_2 \quad x_3' \quad x_4 \quad x_5' \quad x_6 \quad x_8] \quad (23)$$

$$T_1' T_2' T_3' T_4' T_5' T_6' T_7' T_8' T_9 = \begin{bmatrix} x_2 & x_3' & x_4 & x_5 & x_6' & x_7 & x_8 \\ x_2 & x_3' & x_4 & x_5' & x_6' & x_7 & x_8 \\ x_2 & x_3' & x_4 & x_5' & x_6 & x_7 & x_8 \end{bmatrix} \quad (24)$$

$$T_1' T_2' T_3' T_4' T_5' T_6' T_7' T_8' T_9' T_{10} = \begin{bmatrix} x_2 & x_3' & x_4 & x_5 & x_8' & x_9 \\ x_2 & x_3' & x_4 & x_5 & x_8 & x_9 \end{bmatrix} \quad (25)$$

$$T_1' T_2' T_3' T_4' T_5' T_6' T_7' T_8' T_9' T_{10}' T_{11} = 0 \quad (26)$$

$$T_1' T_2' T_3' T_4' T_5' T_6' T_7' T_8' T_9' T_{10}' T_{11}' T_{12} = \begin{bmatrix} x_2 & x_3' & x_4 & x_5' & x_6' & x_7 & x_9 \\ x_2 & x_3' & x_4 & x_5' & x_6 & x_7 & x_9 \\ x_2 & x_3' & x_4 & x_5 & x_6' & x_7 & x_9 \end{bmatrix} \quad (27)$$

using equations (17) through (27), we have from equation (16)

$$f\left(x_2, x_3 --, x_9\right) = \begin{bmatrix} x_3 & x_5 & x_8 & & & & \\ x_3 & x_5' & x_6 & x_8 & & & \\ x_3 & x_5' & x_6' & x_7 & x_8 & & \\ x_3 & x_5 & x_8' & x_9 & & & \\ x_2 & x_3' & x_4 & x_5 & x_8 & & \\ x_2 & x_3' & x_4 & x_5' & x_6 & x_8 & \\ x_2 & x_3' & x_4 & x_5 & x_6' & x_7 & x_8 \\ x_2 & x_3' & x_4 & x_5' & x_6' & x_7 & x_8 \\ x_2 & x_3' & x_4 & x_5' & x_6 & x_7 & x_8 \\ x_2 & x_3' & x_4 & x_5 & x_8' & x_9 & \\ x_2 & x_3' & x_4 & x_5 & x_8 & x_9 & \\ x_2 & x_3' & x_4 & x_5' & x_6' & x_7 & x_9 \\ x_2 & x_3' & x_4 & x_5' & x_6 & x_7 & x_9 \\ x_2 & x_3' & x_4 & x_5 & x_6' & x_7 & x_9 \end{bmatrix} \quad (28)$$

In view of equation (28) equation (2) becomes

$$F\left(x_1, x_2 --, x_9\right) = \begin{bmatrix} x_1 & x_3 & x_5 & x_8 & & & & \\ x_1 & x_3 & x_5' & x_6 & x_8 & & & \\ x_1 & x_3 & x_5' & x_6' & x_7 & x_8 & & \\ x_1 & x_3 & x_5 & x_8' & x_9 & & & \\ x_1 & x_2 & x_3' & x_4 & x_5 & x_8 & & \\ x_1 & x_2 & x_3' & x_4 & x_5' & x_6 & x_8 & \\ x_1 & x_2 & x_3' & x_4 & x_5 & x_6' & x_7 & x_8 \\ x_1 & x_2 & x_3' & x_4 & x_5' & x_6' & x_7 & x_8 \\ x_1 & x_2 & x_3' & x_4 & x_5' & x_6 & x_7 & x_8 \\ x_1 & x_2 & x_3' & x_4 & x_5 & x_8' & x_9 & \\ x_1 & x_2 & x_3' & x_4 & x_5 & x_8 & x_9 & \\ x_1 & x_2 & x_3' & x_4 & x_5' & x_6' & x_7 & x_9 \\ x_1 & x_2 & x_3' & x_4 & x_5' & x_6 & x_7 & x_9 \\ x_1 & x_2 & x_3' & x_4 & x_5 & x_6' & x_7 & x_9 \end{bmatrix} \quad (29)$$

Since R. H. S. of equation (29) is the disjunction of pair-wise disjoint conjunctions. Therefore, reliability of refrigeration plant is given by:

$$\begin{aligned} R_S &= P_r\{F(x_1, x_2 ---------- x_9) = 1\} \\ &= R_1 [R_3R_5R_8 + R_3R_6R_8(1-R_5) + R_3R_7R_8(1-R_5)(1-R_6) + R_3R_5R_9(1-R_8) \\ &+ R_2R_4R_5R_8(1-R_3) + R_2R_4R_6R_8(1-R_3)(1-R_5) + R_2R_4R_5R_8R_9(1-R_3) \\ &+ R_2R_4R_5R_9(1-R_3)(1-R_8) + R_2R_4R_7R_8(1-R_3)(1-R_5)(1-R_6) \\ &+ R_2R_4R_5R_7R_8(1-R_3)(1-R_6) + R_2R_4R_6R_7R_8(1-R_3)(1-R_5) \\ &+ R_2R_4R_7R_9(1-R_3)(1-R_5)(1-R_6) + R_2R_4R_6R_7R_9(1-R_3)(1-R_5) \\ &+ R_2R_4R_5R_7R_9(1-R_3)(1-R_6)] \end{aligned} \quad (30)$$

where, R_1, R_2------- R_9 are the reliabilities of the components of the complex system corresponding to the states x_1, x_2------- x_9, respectively .

6. SOME PARTICULAR CASES

6.1. If the reliability of every component of complex system is R

The equation (30) yields:

$$R_S = 2R^8 - R^7 - 7R^6 + 6R^5 + R^4 \quad (31)$$

6.2. When failure rates follow Weibull distribution

Let failure rate of every component of the complex system be *a*, then the reliability of complex system at an instant ‘t’, is given by

$R_{SW}(t) = 2e^{-8at^p} - e^{-7at^p} - 7e^{-6at^p} + 6e^{-5at^p} + e^{-4at^p}$ (32)

here, *p* is a positive parameter.

6.3. When failure rates follow exponential time distribution

Exponential distribution is nothing but a particular case of Weibull distribution for *p=1* and is very useful in various practical problems. Therefore, the reliability of the complex system as a whole at an instant 't', is given by

$R_{SE}(t) = 2e^{-8at} - e^{-7at} - 7e^{-6at} + 6e^{-5at} + e^{-4at}$ (33)

Also, it is interesting to note that

$$R_{SE}(0) = 1$$

Again, the expression for M.T.T.F., in this case is given by:

$$\text{M.T.T.F.} = \int_0^\infty R_{SE}(t)dt$$

$$= \frac{1}{a}\left[\frac{1}{4} - \frac{1}{7} - \frac{7}{6} + \frac{6}{5} + \frac{1}{4}\right]$$

$$= \frac{0.390476}{a} \quad (34)$$

7. NUMERICAL ILLUSTRATION

Setting a=0.001, p=2, t=0, 1,2 ------ in equations (32) and (33) . Also, a=0.001, 0.002-----, in equation (34), one can draw the graphs as given in Figure 2 and Figure 3, respectively.

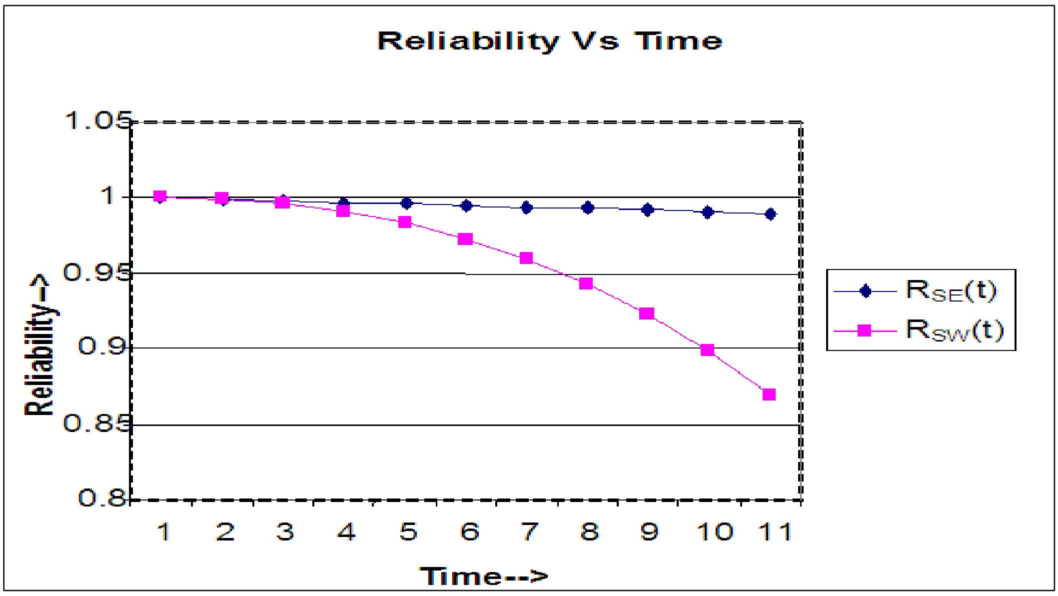

Figure 2: Reliability Vs Time when failures follow Weibull and Exponential time distributions

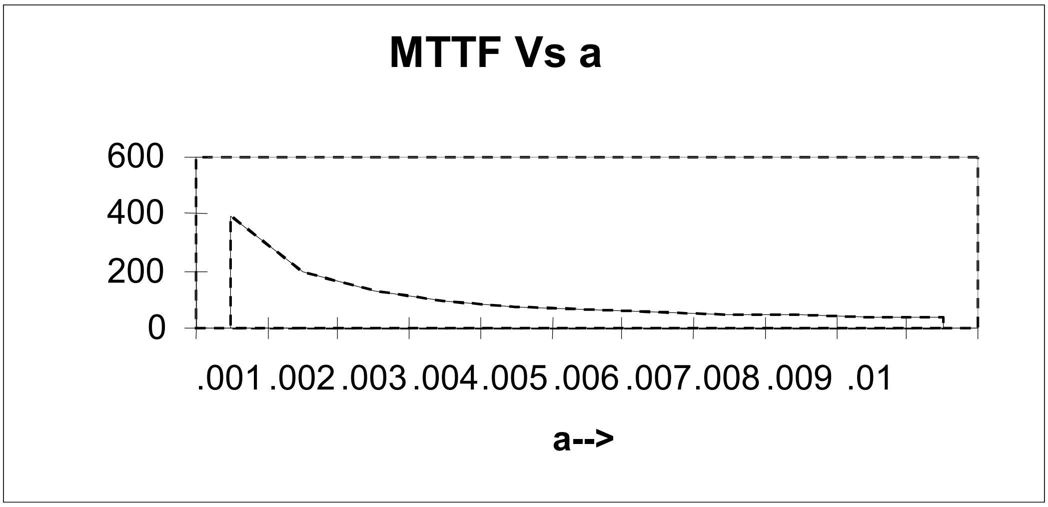

Figure 3: Represents the variation in the values of MTTF

8. CONCLUSION

Table 1 and Figure 2, show that reliability of the complex system decreases in uniform manner in case of exponential distribution.

Table 2 and Figure 3, shows that M.T.T.F. decreases catastrophically in the beginning but thereafter it decreases in a smooth way as we make increase in the failure rate **a**.

REFERENCES

[1] Barlow, R.E and Proschan, F.; “Mathematical Theory of Reliability”, New York, John Wiley, 1965.

[2] Chung, W.K.; “A K-out-of-n:G redundant system with dependant failure rates and common cause failures”, Microelectron. Reliability U.K., Vol 28, pp. 201-203, 1988.

[3] Gnedenko, B.V, Belayer, Y.K and Soloyar; “Mathematical Methods of Reliability Theory”, Academic press, New York, 1969.

[4] Gupta, P.P. and Gupta, R.K.; “Cost analysis of an electronic repairable redundant system with critical human errors”, Microelectron . Reliab. , U.K, Vol 26, pp. 417-421, 1986.

[5] Nagraja, H.N., Kannan, N. and Krishnan, N.B.; “Reliability”, Springer Publication, 2004.

[6] Pandey, D. and Jacob, Mendus; “Cost analysis, availability and MTTF of a three state standby complex system under common-cause and human failures”, Microelectronic. Reliab., U.K., Vol. 35, pp. 91-95, 1995.

[7] Sharma, S.K., Sharma, Deepankar and Masood, Monis; “Availability estimation of urea manufacturing fertilizer plant with imperfect switching and environmental failure”, Journal of Combinatorics, Information & System Sciences, Vol. 29(1-4), pp. 135-141, 2005.

[8] Sharma, Deepankar and Sharma, Jyoti; “Estimation of reliability parameters for telecommunication system”, Journal of combinatorics, information & system sciences, Vol. 29(1-4), pp.151-160, 2005.