Analysis of Working Capabilities for Solar PV System Incorporating Environmental Effects

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In this paper, the author has considered a solar photovoltaic (PV) system to measure its availability as well as profit. Since, the system under consideration is of Non-Markovian nature, the supplementary variables technique has used to formulate a mathematical model. All the failures follow exponential time distribution whereas all repairs follow general time distribution. Pre-emptive repeat policy has been adopted for repair purpose. Availability and the profit function for the solar system have been computed. Long–run flow-state probabilities and a particular case, when repairs follow general time distribution, have also been obtained to fulfill the approach to better study of the system. The analysis of results has been drawn through a graphical illustration followed by a numerical computation.

Keywords: Markovian nature, Supplementary variables technique.

1. INTRODUCTION

This solar PV system is used to generate DC/AC power and this power can be used to operate the electric equipments. There are four subsystem (Named here as A, B, C and D) of this solar PV system, connected in series [1]. The block diagram of solar PV system has been shown in Figure 1(a). The subsystem A is a solar panel and it generates the DC power from sunlight. The subsystem B is a charge controller and controllers the charging of batteries. The subsystem C is a battery bank and contains the two units of batteries, namely C_1 and C_2, connected in parallel redundancy [2]. On failure of either of units C_1 and C_2, the whole system works in reduced efficiency state. The subsystem D is a sine wave inverter and converts the DC power into AC power. Finally, this AC power can be used in house to operate electric equipments. This whole system can fail either due to failure of its any subsystem or due to environmental reasons [3], like heavy raining, low light intensity etc. The repairs of subsystem A, B, and D have pre-empted over the repair of unit C_1 of subsystem C. The system has to wait for repair [4], in case of failure of subsystem C, otherwise repair facilities are always available. The flow of states of considered system has been shown in Figure 1(b).

2. ASSUMPTIONS

The following assumptions have been taken care throughout this study:

(a) At t=0, the whole system is operable.

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(b) All failures follow exponential time distribution whereas all repairs follow general time distribution.

(c) Repairs are perfect [5] and always available but the system has to wait for repair in case of subsystem C.

(d) Nothing can fail from a failed state.

(e) Pre-emptive repeat repair policy has been choosen. The repairs of subsystem A, B and D have pre-empted over the repair of unit C_1 of subsystem C.

(f) The whole system can fail due to environmental reasons.

(g) Failures are S-independent.

3. NOMANCLATURE USED

λ_i	:	Failure rate of i^{th} subsystem.
$\lambda_{e_1}, \lambda_{e_2}$	:	Failure rates due to environmental reasons.
β	:	Waiting rate for repair of subsystem C.
$\mu_i(j)\Delta$	:	First order probability that i^{th} repair will be repaired in the time interval $(j, j+\Delta)$, conditioned that it was not repaired up to the time *j*.
$P_0(t)$	:	Pr {System is operable}.
$P_i(j,t)\Delta$	:	Pr {System suffers with i^{th} failure}.Elapsed repair time lies in the interval $(j, j+\Delta)$.
$P_{C_1 i}(j,t)\Delta$	:	Pr {System suffers with i^{th} failure while unit C_1 of subsystem C has already failed}. Elapsed repair time lies in the interval $(j, j+\Delta)$.
$P_E^W(t)$	:	Pr {System is failed due to failure of subsystem C and is waiting for repair.
$P_C^R(r,t)\Delta$	:	Pr {System is failed due to failure of subsystem C and is ready for repair}.Elapsed repair time lies in the interval $(r, r+\Delta)$}.
$S_i(j)$	:	$\mu_i(j)\exp.\{-\int \mu_i(j)dj\}, \forall\ i\ and\ j$
$\overline{P}(s)$	:	Laplace transform of function $P(t)$.
$D_i(j)$	:	$1-\overline{S}_i(j)/j, \forall\ i\ and$ j .
M_i	:	$-\overline{S}'_i(0)$ and is mean time to repair i^{th} failure.
M.T.T.F.	:	Mean time to failure of solar PV system.
G(t)	:	Profit function for solar PV system.

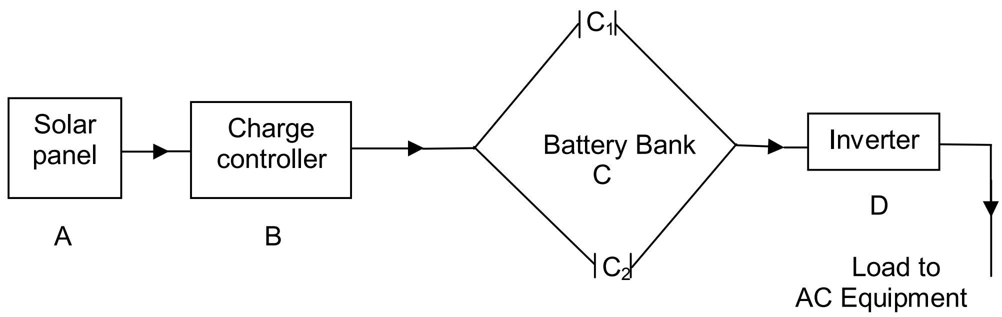


Figure 1 (a): Block-Diagram of Solar PV System

4. FORMULATION OF MATHEMATICAL MODEL

Using elementary probability considerations and limiting procedure [6], we obtain the following set of difference-differential equations [7] corresponding to various flow states depicted in Figure 1(b)

$$\left(\frac{d}{dt}+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_1}+\lambda_{e_1}\right)P_0(t)=\int_0^\infty P_A(x,t)\mu_A(x)dx+\int_0^\infty P_B(y,t)\mu_B(y)dy$$
$$+\int_0^\infty P_D(z,t)\mu_D(z)dz+\int_0^\infty P_{C_1}(m,t)\mu_{C_1}(m)dm$$
$$+\int_0^\infty P_C^R(r,t)\mu_C(r)dr+\int_0^\infty P_{EV}(n,t)\mu_{EV}(n)dn \quad (1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_A(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_B(y)\right]P_B(y,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_D(z)\right]P_D(z,t)=0 \quad (4)$$

(5)

(6)

(7)

(8)

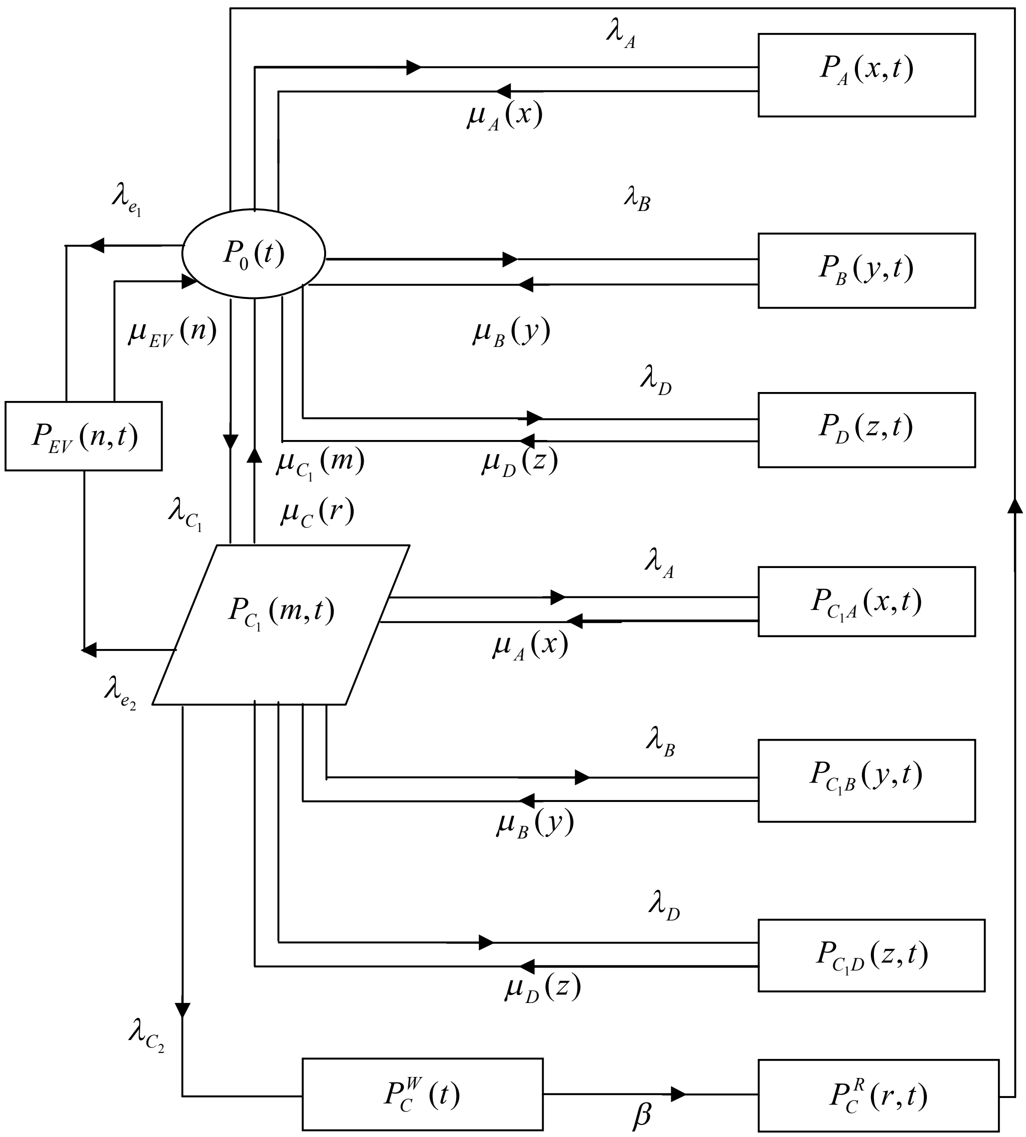
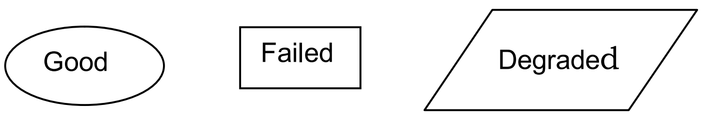


Figure 1(b): Flow of States

States:



$$\left[\frac{d}{dt}+\beta\right]P_C^W(t)=\lambda_{C_2}P_{C_1}(t) \tag{9}$$

$$\left[\frac{\partial}{\partial r}+\frac{\partial}{\partial t}+\mu_C(r)\right]P_C^R(r,t)=0 \tag{10}$$

$$\left[\frac{\partial}{\partial n}+\frac{\partial}{\partial t}+\mu_{EV}(n)\right]P_{EV}(n,t)=0 \tag{11}$$

Boundary conditions are:

$$P_A(0,t)=\lambda_A\ P_0(t) \tag{12}$$

$$P_B(0,t)=\lambda_B\ P_0(t) \tag{13}$$

$$P_D(0,t)=\lambda_D\ P_0(t) \tag{14}$$

$$P_{C_1}(0,t)=\lambda_{C_1}\ P_0(t)+\int_0^\infty P_{C_1A}(x,t)\mu_A(x)dx+\int_0^\infty P_{C_1B}(y,t)\mu_B(y)dy +\int_0^\infty P_{C_1D}(z,t)\mu_D(z)dz \tag{15}$$

$$P_{C_1A}(0,t)=\lambda_A P_{C_1}(t) \tag{16}$$

$$P_{C_1B}(0,t)=\lambda_B P_{C_1}(t) \tag{17}$$

$$P_{C_1D}(0,t)=\lambda_D P_{C_1}(t) \tag{18}$$

$$P_C^R(0,t)=\beta\ P_C^W(t) \tag{19}$$

$$P_{EV}(0,t)=\lambda_{e_1}\ P_0(t)+\lambda_{e_2}P_{C_1}(t) \tag{20}$$

Initial conditions are:

$$P_i(0)=\begin{cases}1, & if\ \ i=0\\ 0, & if\ \ i\neq 0\end{cases} \tag{21}$$

5. SOLUTION OF THE MODEL

Taking Laplace transforms of equations (1) through (20) subjected to initial conditions (21) and then on solving them one by one, we may obtain:

$$\overline{P}_0(s)=\frac{1}{B(s)} \tag{22}$$

$$\overline{P}_A(s)=\frac{\lambda_A}{B(s)}D_A(s) \tag{23}$$

$$\overline{P}_B(s)=\frac{\lambda_B}{B(s)}D_B(s) \tag{24}$$

$$\overline{P}_D(s)=\frac{\lambda_D}{B(s)}D_D(s) \tag{25}$$

$$\overline{P}_{C_1}(s)=\frac{\lambda_{C_1}A(s)}{B(s)} \tag{26}$$

$$\overline{P}_{C_1A}(s)=\frac{\lambda_A\lambda_{C_1}A(s)}{B(s)}D_A(s) \tag{27}$$

$$\overline{P}_{C_1B}(s)=\frac{\lambda_B\lambda_{C_1}A(s)}{B(s)}D_B(s) \tag{28}$$

$$\overline{P}_{C_1D}(s)=\frac{\lambda_D\lambda_{C_1}A(s)}{B(s)}D_D(s) \tag{29}$$

$$\overline{P}_C^W(s)=\frac{\lambda_{C_2}\lambda_{C_1}A(s)}{(s+\beta)B(s)} \tag{30}$$

$$\overline{P}_C^R(s)=\frac{\beta\lambda_{C_1}\lambda_{C_2}A(s)}{(s+\beta)B(s)}D_C(s) \tag{31}$$

and $\overline{P}_{EV}(s)=\frac{1}{B(s)}\left[\lambda_{e_1}+\lambda_{e_2}\lambda_{C_1}A(s)\right]D_{EV}(s)$ (32)

where, $A(s)=\dfrac{D_{C_1}\left(s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}\right)}{1-\left[\lambda_A\overline{S}_A(s)+\lambda_B\overline{S}_B(s)+\lambda_D\overline{S}_D(s)\right]D_{C_1}\left(s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}\right)}$ (33)

and
$$\begin{aligned}B(s)=&\ s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_1}+\lambda_{e_1}-\lambda_A\overline{S}_A(s)-\lambda_B\overline{S}_B(s)-\lambda_D\overline{S}_D(s)\\&-\frac{\beta}{s+\beta}\lambda_{C_1}\lambda_{C_2}A(s)\overline{S}_C(s)-\left[\lambda_{e_1}+\lambda_{e_2}\lambda_{C_1}A(s)\right]\overline{S}_{EV}(s)\\&-\lambda_{C_1}\left[1+\left\{\lambda_A\overline{S}_A(s)+\lambda_B\overline{S}_B(s)+\lambda_D\overline{S}_D(s)\right\}A(s)\right]\\&\times\overline{S}_{C_1}\left(s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}\right)\end{aligned} \tag{34}$$

It is worth noticing that

sum of equations (22) through (32) = $\frac{1}{s}$ (35)

6. LONG-RUN FLOW-STATE PROBABILITIES

By using Abel's Lemma, viz., $\underset{t\to\infty}{Lim}\, P(t) = \underset{s\to 0}{Lim}\, s\overline{P}(s) = P$ (say), provided the limit on left exists, we obtain the following long-run flow-state probabilities from equations (22) through (32):

$$P_0 = \frac{1}{B'(0)} \tag{36}$$

$$P_A = \frac{\lambda_A M_A}{B'(0)} \tag{37}$$

$$P_B = \frac{\lambda_B M_B}{B'(0)} \tag{38}$$

$$P_D = \frac{\lambda_D M_D}{B'(0)} \tag{39}$$

$$P_{C_1} = \frac{\lambda_{C_1} A(0)}{B'(0)} \tag{40}$$

$$P_{C_1 A} = \frac{\lambda_A \lambda_{C_1} A(0)}{B'(0)} M_A \tag{41}$$

$$P_{C_1 B} = \frac{\lambda_B \lambda_{C_1} A(0)}{B'(0)} M_B \tag{42}$$

$$P_{C_1 D} = \frac{\lambda_D \lambda_{C_1} A(0)}{B'(0)} M_D \tag{43}$$

$$P_C^W = \frac{\lambda_{C_1} \lambda_{C_2} A(0)}{\beta . B'(0)} \tag{44}$$

$$P_C^R = \frac{\lambda_{C_1} \lambda_{C_2} A(0)}{B'(0)} M_C \tag{45}$$

and $P_{EV} = \frac{1}{B'(0)}\left[\lambda_{e_1} + \lambda_{e_2}\lambda_{C_1} A(0)\right] M_{EV}$ (46)

where, $A(0) = \dfrac{1 - \overline{S}_{C_1}\left(\lambda_A + \lambda_B + \lambda_D + \lambda_{C_2} + \lambda_{e_2}\right)}{\lambda_{C_2} + \lambda_{e_2} + \left(\lambda_A + \lambda_B + \lambda_D\right)\overline{S}_{C_1}\left(\lambda_A + \lambda_B + \lambda_D + \lambda_{C_2} + \lambda_{e_2}\right)}$ (47)

$M_i = -\overline{S}'_i(0) =$ Mean time to repair i^{th} failure

and $B'(0) = \left[\frac{d}{ds} B(s)\right]_{s=0}$

7. A PARTICULAR CASE

7.1. When all repairs follow exponential time distribution

In this case, we may obtain the following Laplace transforms of various flow-state probabilities from equations (22) through (32) by substituting $\overline{S}_i(s) = \frac{\mu_i}{s+\mu_i}$, for all *i* and s:

$$\overline{P}_0(s) = \frac{1}{E(s)} \tag{48}$$

$$\overline{P}_A(s) = \frac{\lambda_A}{E(s)} \cdot \frac{1}{s+\mu_A} \tag{49}$$

$$\overline{P}_B(s) = \frac{\lambda_B}{E(s)} \cdot \frac{1}{s+\mu_B} \tag{50}$$

$$\overline{P}_D(s) = \frac{\lambda_D}{E(s)} \cdot \frac{1}{s+\mu_D} \tag{51}$$

$$\overline{P}_{C_1}(s) = \frac{\lambda_{C_1} C(s)}{E(s)} \tag{52}$$

$$\overline{P}_{C_1A}(s) = \frac{\lambda_A \lambda_{C_1} C(s)}{E(s)} \cdot \frac{1}{s+\mu_A} \tag{53}$$

$$\overline{P}_{C_1B}(s) = \frac{\lambda_B \lambda_{C_1} C(s)}{E(s)} \cdot \frac{1}{s+\mu_B} \tag{54}$$

$$\overline{P}_{C_1D}(s) = \frac{\lambda_D \lambda_{C_1} C(s)}{E(s)} \cdot \frac{1}{s+\mu_D} \tag{55}$$

$$\overline{P}_C^W(s) = \frac{\lambda_{C_1} \lambda_{C_2} C(s)}{(s+\beta)E(s)} \tag{56}$$

$$\overline{P}_C^R(s) = \frac{\beta \lambda_{C_1} \lambda_{C_2} C(s)}{(s+\beta)E(s)} \cdot \frac{1}{s+\mu_C} \tag{57}$$

and $\overline{P}_{EV}(s) = \frac{1}{E(\mathrm{s})}\left[\lambda_{e_1} + \lambda_{e_2}\lambda_{C_1} C(s)\right] \frac{1}{s+\mu_{EV}}$ (58)

where, $C(s) = \dfrac{1}{s\left[1 + \frac{\lambda_A}{s+\mu_A} + \frac{\lambda_B}{s+\mu_B} + \frac{\lambda_D}{s+\mu_D}\right] + \lambda_{C_2} + \lambda_{e_2} + \mu_{C_1}}$ (59)

and $$E(s) = s\left[1 + \frac{\lambda_A}{s+\mu_A} + \frac{\lambda_B}{s+\mu_B} + \frac{\lambda_D}{s+\mu_D}\right] + \lambda_{C_1} + \lambda_{e_1}$$

$$- \frac{\beta\lambda_{C_1}\lambda_{C_2}C(s)\mu_C}{(s+\beta)(s+\mu_C)}\left[\lambda_{e_1} + \lambda_{e_2}\lambda_{C_1}C(s)\right]\frac{\mu_{EV}}{s+\mu_{EV}}$$

$$-\left[1 + \left\{\frac{\lambda_A\mu_A}{s+\mu_A} + \frac{\lambda_B\mu_B}{s+\mu_B} + \frac{\lambda_D\mu_D}{s+\mu_D}\right\}C(s)\right]\frac{\lambda_{C_1}\mu_{C_1}}{(s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}+\mu_{C_1}} \quad (60)$$

8. AVAILABILITY EVALUATION

We have,

$$\overline{P}_{up}(\mathrm{s}) = \frac{1}{s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_1}+\lambda_{e_1}}\left[1 + \frac{\lambda_{C_1}}{s+\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}}\right]$$

Taking inverse Laplace transform, we obtain availability of the considered solar PV system as:

$$P_{up}(t) = (1+F)\exp.\left\{-\left(\lambda_A+\lambda_B+\lambda_D+\lambda_{C_1}+\lambda_{e_1}\right)t\right\}$$
$$- F\exp.\left\{-\left(\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}\right)t\right\} \quad (61)$$

where, $$F = \frac{\lambda_{C_1}}{\lambda_{C_2}+\lambda_{e_2}-\lambda_{C_1}-\lambda_{e_1}} \quad (62)$$

Note that $P_{up}(0) = 1$

Also,

$$P_{down}(t) = 1 - P_{up}(t) \quad (63)$$

9. PROFIT FUNCTION

Profit function for the considered system is given by

$$G(t) = C_1\int_0^t P_{up}(t)dt - C_2 t$$

where, C_1 and C_2 are the revenue and repair cost per unit time.

$$\therefore G(t) = C_1\left[(1+F).\frac{1-e^{-\left(\lambda_A+\lambda_B+\lambda_D+\lambda_{C_1}+\lambda_{e_1}\right)t}}{\left(\lambda_A+\lambda_B+\lambda_D+\lambda_{C_1}+\lambda_{e_1}\right)}\right.$$

$$- F.\frac{1-e^{-\left(\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}\right)t}}{\left(\lambda_A+\lambda_B+\lambda_D+\lambda_{C_2}+\lambda_{e_2}\right)}\Bigg] - C_2 t \quad (64)$$

where, the value of F has been mentioned earlier in equation (62).

10. NUMERICAL ILLUSTRATION

For a numerical illustration, let us consider the values:

$\lambda_A = 0.05,\ \lambda_B = 0.07,\ \lambda_D = 0.06,\ \lambda_{C_1} = 0.001,\ \lambda_{C_2} = 0.002,\ \lambda_{e_1} = 0.003,$ $\lambda_{e_2} = 0.004,\ C_1 = Rs.6.00,\ C_2 = Rs.2.00$ and t = 0, 1, 2, -----.

By using these values in equations (61) and (64), we have drawn the Figure 2 and Figure 3.

11. RESULTS AND DISCUSSION

The author has been computed the value of availability function and its graph has been shown in Figure 2. This Figure 2 shows that availability of considered system decreases rapidly initially but after that it decreases approximately in constant manner.

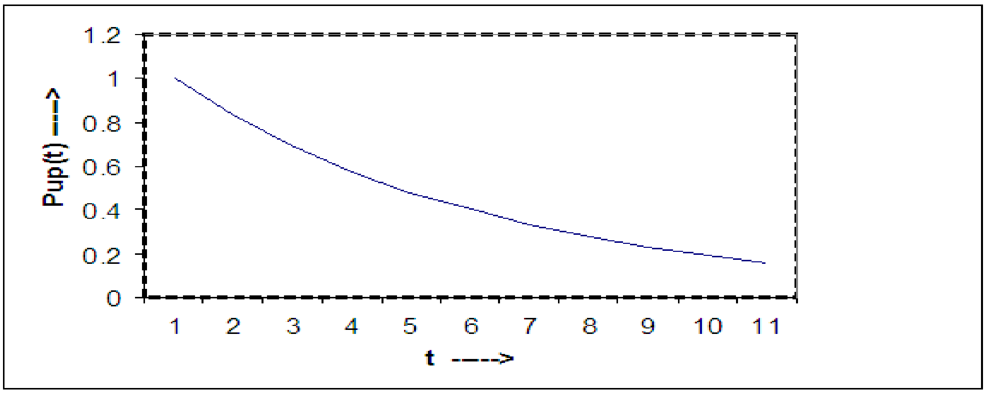


Figure 2: Shows availability of the system at various times.

The value of profit function for different values of time t has been shown through Figure 3. This yields the conclusion that up to t= 6, value of profit function increases and the maximum value of profit function is 9.850940. After t=6 it again starts decreasing smoothly.

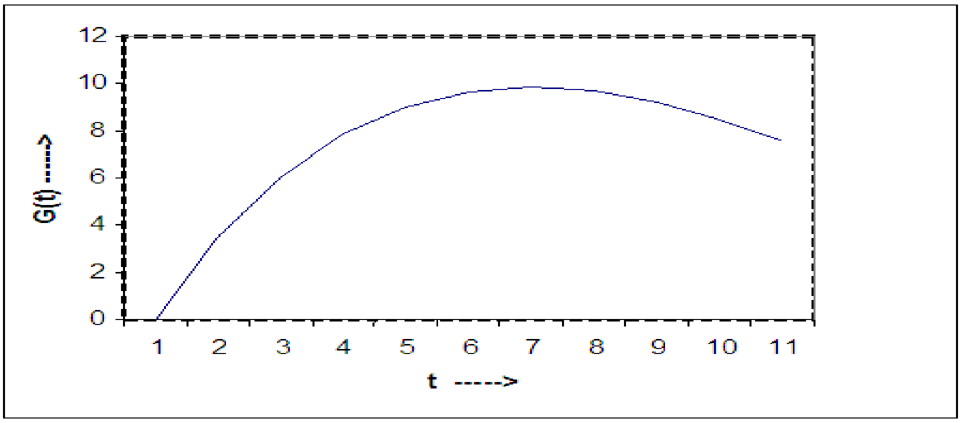


Figure 3: Shows profit function of the system at various times.

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