Performance Analysis for a Network having Standby Redundant Unit with Waiting in Repair

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In this paper, the authors deal with a complex network having n-identical units in series and two identical standby units for evaluation of its ability measures. The authors have been used supplementary variables technique to formulate the mathematical equations for various flow-states. These mathematical equations have been solved by the application of Laplace transform. The Laplace transforms of various flow-states probabilities have obtained. We have calculated the expressions for reliability function, availability function and mean time to failure (M.T.T.F.). Analysis of time-independent state probabilities and a particular case, when all repairs follow exponential time distribution, have also given to improve the practical utility of the model.

Keywords: Computer network, reliability analysis, supplementary variables, Laplace transform etc.

1. INTRODUCTION

The whole system consists of two subsystems *A* and *B*, connected in series. The subsystem *B* has two identical units B_1 and B_2 in standby redundancy [1]. We can online the units of subsystem *B* by the application of imperfect switching device [2]. The system configuration has shown in Figure 1(a). The picture drawn in Figure 1(b) shows the flow of states. The whole system may fail due to failure of its either subsystems. The system can be repaired immediately [5] if the subsystem *A* or the unit B_1 of subsystem *B* or both are failed. The system has to wait for repair in case of repair of whole subsystem *B*.

We may use the results obtained in this study to every network system of real life, with similar configurations. A numerical example together with its graphical illustration has also appended at the end to highlight important results of the study.

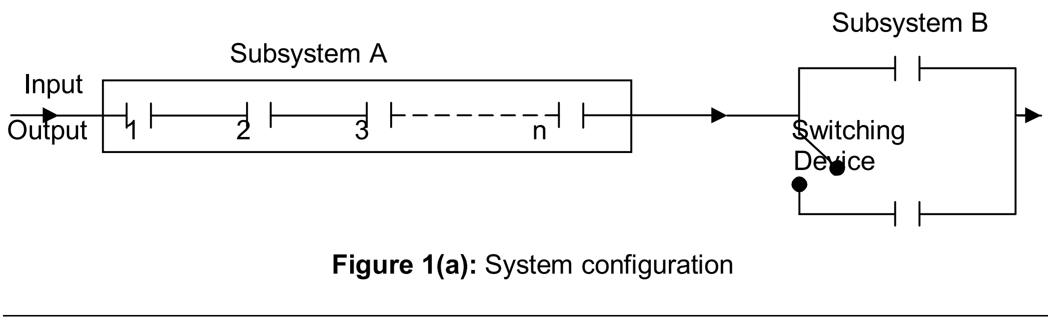

Figure 1(a): System configuration

The following assumptions have been associated with this study:

1. Initially, the whole network is good and operable with full efficiency.
2. There are n units connected in series in subsystem A.
3. There is one standby unit in subsystem B and on failure of main unit, we can online standby unit through imperfect switching device.
4. All failures follow exponential time distribution [3] and are S-independent.
5. All repairs follow general time distribution and are perfect.
6. On failure of both the units of subsystem B, the system has to wait for repair.
7. Repair of any one failed unit of subsystem A can be done immediately.
8. Nothing can fail from a failed state.

List of notations used in this study is as follows:

λ_A / λ_B	:	Failure rate of subsystem A/B.
$(1-\alpha)$	:	Failure rate of switching device.
w	:	Waiting rate for repair of whole subsystem B.
$\mu_i(j)\Delta$	:	First order probability that i^{th} failure can be repaired in the time interval $(j, j+\Delta)$ conditioned that it was not repaired up to the time *j*.
$P_{0,0,0}(t)$	:	Pr {at time *t*, subsystem A, B and switching device is operable i.e. the whole system is operable}.
$P_{0,B_1,0}(t)$	:	Pr {at time t, system is operable while one online unit of subsystem B has failed.
$P^{W}_{0,F,0}(t)$	:	Pr {at time t, system is failed due to failure of subsystem B and is waiting for repair}.
$P^{R}_{0,F,0}(z,t).\Delta$	:	Pr (at time t, system is failed due to failure of subsystem B and is ready to repair}. Elapsed repair time lies in the interval $(z, z+\Delta)$.
$P_{F,0,0}(x,t)\Delta etc.$	:	Pr (at time t, system is failed due to failure of subsystem A}. Elapsed repair time lies in the interval $(x, x+\Delta)$.
$S_i(j)$	:	$\mu_i(j)\exp.\left\{-\int \mu_i(j)dj\right\} \quad \forall\, i$ and j.
$\overline{P}(s)$	:	Laplace transform (L.T.) of function P (t).
$D_i(j)$	:	$1-\overline{S}_i(j)/j$, $\forall\, i$ and j.

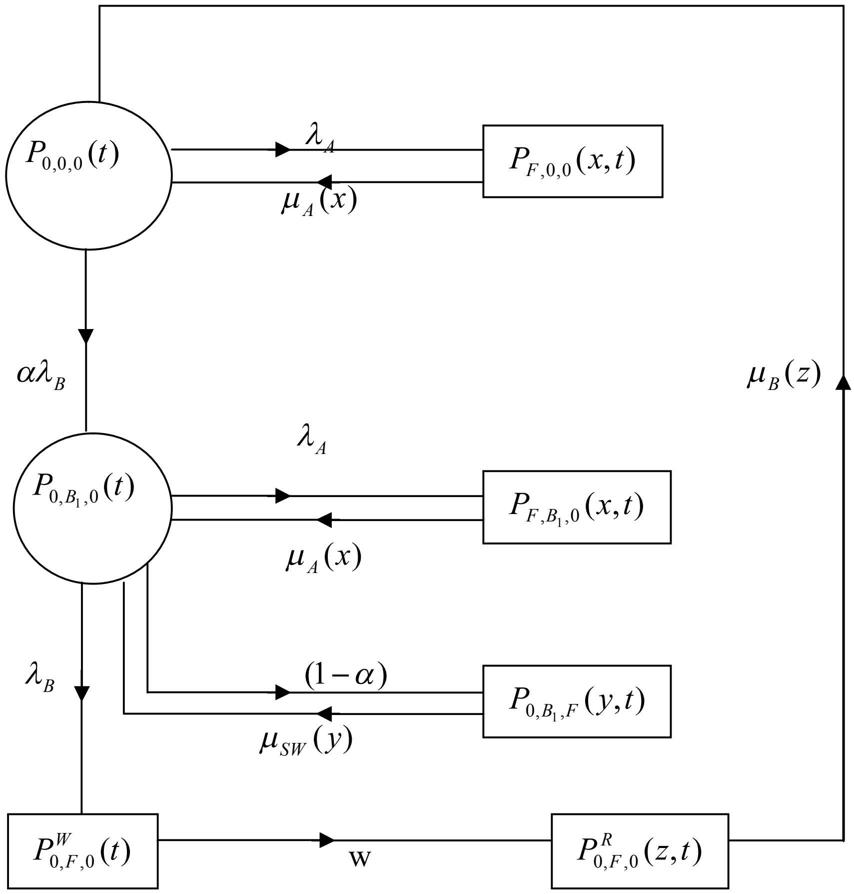

Figure 1(b): State-transition diagram

2. LITERATURE REVIEW

In this section, author has done analysis for mathematical formulation of the model, its solution, some particular cases and various results related to reliability estimation.

2.1. Formulation of mathematical model

By using continuity argument and limiting procedure [5], we obtain the following set of difference-differential equations, which is continuous in time and discrete in space [6], governing the behavior of considered model

$$\left[\frac{d}{dt}+\lambda_A+\alpha\lambda_B\right]P_{0,0,0}(t)=\int_0^\infty P_{F,0,0}(x,t)\mu_A(x)dx+\int_0^\infty P^R_{0,F,0}(z,t)\mu_B(z)dz \quad (1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_{F,0,0}(x,t)=0 \tag{2}$$

$$\left[\frac{d}{dt}+\lambda_A+(1-\alpha)+\lambda_B\right]P_{0,B_1,0}(t)=\alpha\lambda_B P_{0,0,0}(t)+\int_0^\infty P_{F,B_1,0}(x,t)\mu_A(x)dx + \int_0^\infty P_{0,B_1,F}(y,t)\mu_{SW}(y)dy \tag{3}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_{F,B_1,0}(x,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_{SW}(y)\right]P_{0,B_1,F}(y,t)=0 \tag{5}$$

$$\left[\frac{d}{dt}+w\right]P^W_{0,F,0}(t)=\lambda_B \quad P_{0,B_1,0}(t) \tag{6}$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_B(z)\right]P^R_{0,F,0}(z,t)=0 \tag{7}$$

Boundary conditions are

$$P_{F,0,0}(0,t)=\lambda_A\ P_{0,0,0}(t) \tag{8}$$

$$P_{F,B_1,0}(0,t)=\lambda_A\ P_{0,B_1,0}(t) \tag{9}$$

$$P_{0,B_1,F}(0,t)=(1-\alpha)\ P_{0,B_1,0}(t) \tag{10}$$

$$P^R_{0,F,0}(0,t)=w\,P^W_{0,F,0}(t) \tag{11}$$

Initial conditions are

$P_{0,0,0}(0)=1$, all other state probabilities are zero at $t=0$. (12)

2.2. Solution of the model

We shall solve the above system of difference-differential equations with the aid of Laplace transform to obtain probabilities of different transition states, depicted in fig-1. Thus, taking Laplace transform [7,8], of equations (1) through (11) subjected to initial conditions (12) and then on solving them one by one, we obtain

$$\overline{P}_{0,0,0}(s)=\frac{1}{B(s)} \tag{13}$$

$$\overline{P}_{F,0,0}(s)=\frac{\lambda_A D_A(s)}{B(s)} \tag{14}$$

$$\overline{P}_{0,B_1,0}(s) = \frac{A(s)}{B(s)} \tag{15}$$

$$\overline{P}_{F,B_1,0}(s) = \frac{\lambda_A D_A(s)\,\mathrm{A}(s)}{B(s)} \tag{16}$$

$$\overline{P}_{0,B_1,F}(s) = \frac{(1-\alpha)D_{SW}(s)\,\mathrm{A}(s)}{B(s)} \tag{17}$$

$$\overline{P}^{W}_{0,F,0}(s) = \frac{\lambda_B\,\mathrm{A}(s)}{B(s)(s+w)} \tag{18}$$

$$\overline{P}^{R}_{0,F,0}(s) = \frac{\lambda_B \mathrm{w}\, D_B(s) A(s)}{B(s)(s+w)} \tag{19}$$

where $A(s) = \dfrac{\alpha\lambda_B}{s\left[1+\lambda_A D_A(s)+(1-\alpha)D_{SW}(s)\right]+\lambda_B}$ (20)

and $B(s) = s+\lambda_A+\alpha\lambda_B-\lambda_A\overline{S}_A(s)-A(s)\overline{S}_B(s)\dfrac{\lambda_B w}{s+w}$ (21)

and $B(s) = s+\lambda_A+\alpha\lambda_B-\lambda_A\overline{S}_A(s)-A(s)\overline{S}_B(s)\dfrac{\lambda_B w}{s+w}$ (22)

It is interesting to note here that Sum of equations (13) through (19) = $\dfrac{1}{s}$ (23)

2.3. Long-run behaviour of the system

Using final value theorem in L.T., *viz*; $\lim_{t\to\infty} P(t) = \lim_{s\to 0} s\overline{P}(s) = P(say)$, provided limit on LHS exists, in equations (13) through (19) we obtain the following long-run behavior [9,10] of the considered system:

$$P_{0,0,0} = \frac{1}{B'(0)} \tag{24}$$

$$P_{F,0,0} = \frac{\lambda_{\mathrm{A}} M_A}{B'(0)} \tag{25}$$

$$P_{0,B_1,0} = \frac{\alpha}{B'(0)} \tag{26}$$

$$P_{F,B_1,0} = \frac{\lambda_A \alpha M_A}{B'(0)} \tag{27}$$

$$P_{0,B_1,F} = \frac{\alpha(1-\alpha)M_{SW}}{B'(0)} \tag{28}$$

$$P^{W}_{0,F,0} = \frac{\alpha\lambda_B}{wB'(0)} \tag{29}$$

$$P^{R}_{0,F,0} = \frac{\alpha\lambda_B M_B}{B'(0)} \tag{30}$$

where $M_i = -\overline{S}'_i(0) =$ Mean time to repair subsystem *i*

and $B'(0) = \left[\frac{d}{ds}B(s)\right]_{s=0}$

2.4. Some particular case

Case (i): When all repairs follow exponential time distribution.

In this case, setting $\overline{S}_i(j) = \mu_i \big/ (j+\mu_i)$, $\forall$ *i* and *j*, in equations (13) through (19), we obtain the following L.T. of transition-state probabilities:

$$\overline{P}_{0,0,0}(s) = \frac{1}{E(s)} \tag{31}$$

$$\overline{P}_{F,0,0}(s) = \frac{\lambda_A}{E(s)(s+\mu_A)} \tag{32}$$

$$\overline{P}_{0,B_1,0}(s) = \frac{C(s)}{E(s)} \tag{33}$$

$$\overline{P}_{F,B_1,0}(s) = \frac{\lambda_A C(s)}{E(s)(s+\mu_A)} \tag{34}$$

$$\overline{P}_{0,B_1,F}(s) = \frac{(1-\alpha)C(s)}{E(s)(s+\mu_{SW})} \tag{35}$$

$$\overline{P}^{W}_{0,F,0}(s) = \frac{\lambda_B C(s)}{E(s)(s+w)} \tag{36}$$

$$\overline{P}^{R}_{0,F,0}(s) = \frac{\lambda_B w C(s)}{E(s)(s+w)(s+\mu_B)} \tag{37}$$

where $$C(s) = \frac{\alpha\lambda_B}{s\left[1+\frac{\lambda_A}{s+\mu_A}+\frac{(1-\alpha)}{s+\mu_{SW}}\right]+\lambda_B} \tag{38}$$

and $$E(s) = s+\alpha\lambda_B+\frac{s\lambda_A}{s+\mu_A}-C(s)\frac{\lambda_B w\mu_B}{(s+w)(s+\mu_B)} \tag{39}$$

Case (ii): When switching device used, is perfect.

In this case, putting $\alpha = 1$ in equations (13) to (19), we obtain the required results. Note that in this case, $\overline{P}_{0,B_1,F}(s) = 0$.

2.5. Reliability and MTTF of the system

Reliability of the considered network can be obtained as

$$\overline{R}(s) = \frac{1}{s + \lambda_A + \alpha\lambda_B}$$

or, $$R(t) = L^{-1}\left\{\overline{R}(s)\right\}$$

$$\therefore R(t) = \exp\left\{-\left(\lambda_A + \alpha\lambda_B\right)\mathrm{t}\right\} \tag{40}$$

Also, $$M.T.T.F. = \int_0^\infty R(t)dt$$

$$= \frac{1}{\lambda_A + \alpha\lambda_B} \tag{41}$$

2.6. Availability of the system

For the considered network

$$\overline{P}_{up}(s) = \frac{1}{s + \lambda_A + \alpha\lambda_B}\left(1 + \frac{\alpha\lambda_B}{s + \lambda_A + (1-\alpha) + \lambda_B}\right)$$

$$\therefore P_{up}(t) = \left[1 + \frac{\alpha\lambda_B}{(1-\alpha)(1+\lambda_B)}\right]e^{-(\lambda_A + \alpha\lambda_B)\,t} - \frac{\alpha\lambda_B}{(1-\alpha)(1+\lambda_B)}e^{-(\lambda_A + (1-\alpha) + \lambda_B)\,t} \tag{42}$$

Also, $P_{down}(t) = 1 - P_{up}(t)$

2.7 Numerical illustration

For a numerical illustration, let us consider the following value set for parameters:

$\lambda_A = 0.45,\ \lambda_B = 0.37,\ \alpha = 0.7$ and t = 0,1,2,-----10.

3. RESULTS AND DISCUSSION

Table 1 gives the values of reliability of considered system for various values of time *t.* Its graph has been shown in Figure 2. Analysis of Table 1 and Figure 2 reveal that the reliability of considered system decreases approximately in constant manner and there are no sudden jumps in the values of reliability.

Table 1: Values of reliability of considered system for various values of time *t*.

t	R(t)
0	1
1	0.492136
2	0.242198
3	0.119194
4	0.05866
5	0.028869
6	0.014207
7	0.006992
8	0.003441
9	0.001693
10	0.000833

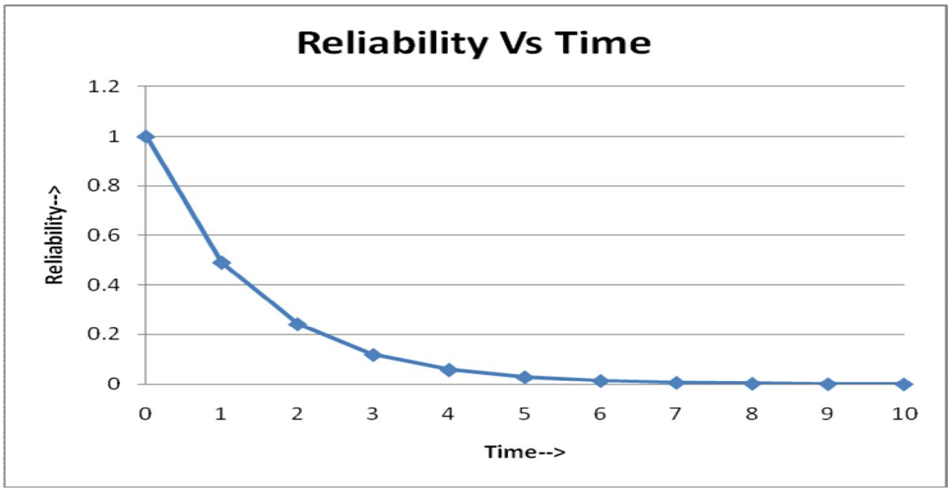

Figure 2: Reliability of considered system for various values of time *t*

Table 2 gives the values of availability of considered system for different values of time *t*. Its graph has been shown in Figure 3. Critical examination of Table 2 and Figure 3 yield that value of availability decreases rapidly in the beginning but thereafter it decreases constantly.

Table 2: Values of availability of considered system for different values of time *t*.

t	Pup(t)
0	1
1	0.596659
2	0.327741
3	0.17242
4	0.088485
5	0.044731
6	0.0224
7	0.01115
8	0.005529
9	0.002734
10	0.00135

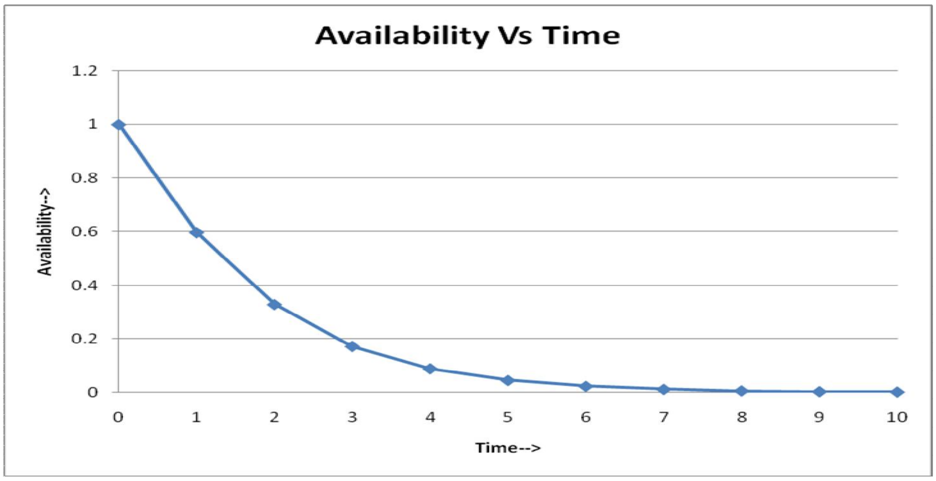

Figure 3: Values of availability of considered system for different values of time *t*.

Table 3 gives the values of M.T.T.F. of considered system for different values of failure rate of subsystem *A*. Its graph has been shown in Figure 4. Analysis of Table 3 and Figure 4 yield that value of M.T.T.F. decreases catastrophically.

Table 4: Values of M.T.T.F. of considered system for different values of failure rate of subsystem *A.*

λ_A	**MTTF**
0	3.861004
0.01	3.717472
0.02	3.584229
0.03	3.460208
0.04	3.344482
0.05	3.236246
0.06	3.134796
0.07	3.039514
0.08	2.949853
0.09	2.86533
0.1	2.785515

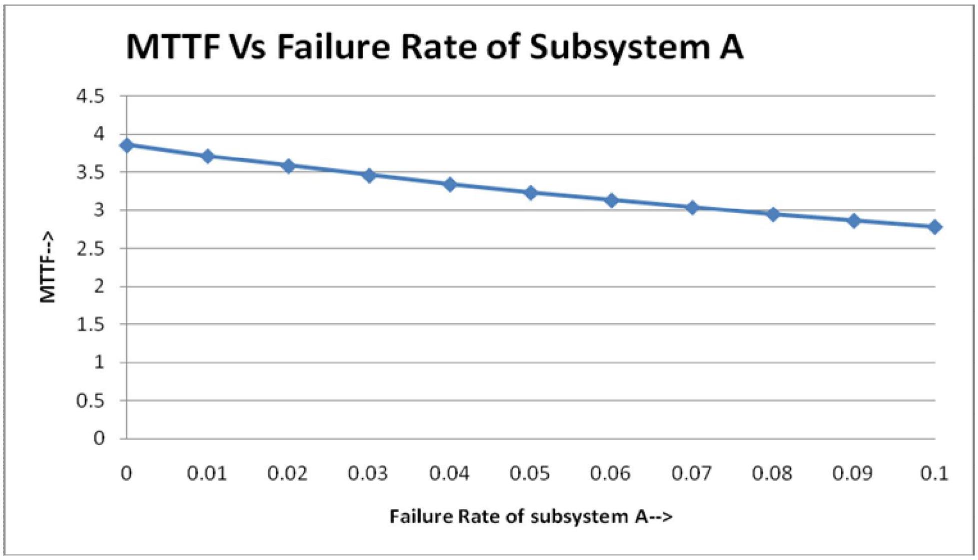

Figure 4: Values of M.T.T.F. of considered system for different values of failure rate of subsystem *A.*

Table 4 gives the values of M.T.T.F. of considered system for different values of failure rate of subsystem *B*. Its graph has been shown in Figure 5. Analysis of Table 4 and Figure 5 yield that value of M.T.T.F. decreases catastrophically.

Table 4: Values of M.T.T.F. of considered system for different values of failure rate of subsystem *B.*

λ_B	MTTF
0	2.222222
0.001	2.218771
0.002	2.21533
0.003	2.2119
0.004	2.208481
0.005	2.205072
0.006	2.201673
0.007	2.198285
0.008	2.194908
0.009	2.191541
0.01	2.188184

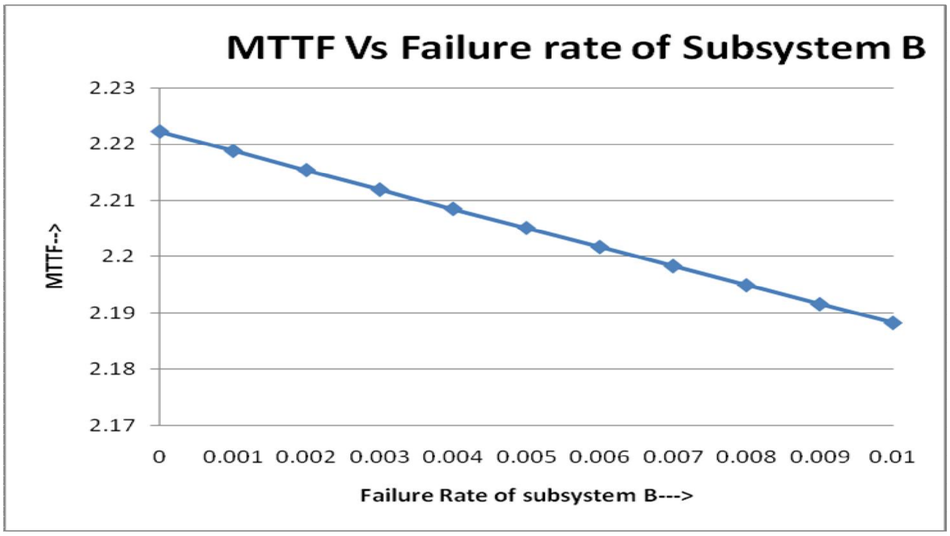

Figure 4: Values of M.T.T.F. of considered system for different values of failure rate of subsystem *B.*

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