Generalized Theorem On Multivariable H-Function Transform

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In this research paper, we obtain an interesting theorem involving multivariable H-function transform. This theorem is quite general in nature and capable of giving a number of new, interesting and useful results. The results obtained by Chaurasia and Gupta are special cases of main finding.

Keywords: H-function transform.

1. INTRODUCTION

In this paper, we define the multivariable H-function transform of a function $f(x_1, ..., x_r)$ by the following integral:

$$H[f(x_1,...,x_r); s_1,...,s_r] = \int_0^\infty ... \int_0^\infty H^{0,0:\{M_r, N_r\}}_{P,Q:\{P_r,Q_r\}} \left[\begin{matrix} s_1 x_1 \\ \vdots \\ s_r x_r \end{matrix} \middle| \begin{matrix} (a_j; \alpha'_j, ..., \alpha_j^{(r)})_{1,P} : \{(c_j^{(r)}, \gamma_j^{(r)})_{1,P_r}\} \\ (b_j; \beta'_j, ..., \beta_j^{(r)})_{1,Q} : \{(d_j^{(r)}, \delta_j^{(r)})_{1,Q_r}\} \end{matrix} \right] f(x_1,...,x_r)\, dx_1,...,dx_r \tag{1.1}$$

where $\{M_r, N_r\}$ stands for $M_1, N_1; ...; M_r, N_r$ and $\{(c_j^{(r)}, \gamma_j^{(r)})_{1,P_r}\}$ stands for the sequence of r-ordered pairs $(c'_j, \gamma'_j)_{1,P_1}; ...; (c_j^{(r)}, \gamma_j^{(r)})_{1,P_r}$. And where the H-function of r complex variables z_1,...,z_r was introduced by Srivastava and Panda [2]. We shall define and represent it in the following contracted form pp. 251, Eq. (C.1) of Srivastava and Goyal [1] :

$$H^{0,n:\{M_r,N_r\}}_{P,Q:\{P_r,Q_r\}} \left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} \left(a_j; \alpha'_j, ..., \alpha_j^{(r)}\right)_{1,P} : \left\{\left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,P_r}\right\} \\ \left(b_j; \beta'_j, ..., \beta_j^{(r)}\right)_{1,Q} : \left\{\left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,Q_r}\right\} \end{matrix} \right] = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi_1(\xi_1) \cdots \phi_r(\xi_r) \psi(\xi_1 \cdots \xi_r) z_1^{\xi_1} \cdots z_r^{\xi_r} d\xi_1 \cdots d\xi_r$$

where $\omega = \sqrt{-1}$. For the convergence, existence conditions and other details of the above multivariable H-function, we refer to pp. 251-253, Eq.(C.2)-(C.8) from Srivastava et al [1].

The mellin transform of the H-function of two variables is given by pp.147, eq. (8.5.1) [3] as :

$$\int_0^\infty ... \int_0^\infty x^{s-1} y^{t-1} H^{0,0:M_1,N_1;M_2,N_2}_{P,Q:P_1,Q_1;P_2,Q_2} \left[\begin{matrix} ax^\lambda \\ bx^\mu \end{matrix} \middle| \begin{matrix} (a_j; \alpha'_j, \alpha''_j)_{1,P} : (c'_j, \gamma'_j)_{1,P_1}; (c''_j, \gamma''_j)_{1,P_2} \\ (b_j; \beta'_j, \beta''_j)_{1,Q} : (d'_j, \delta'_j)_{1,Q_1}; (d''_j, \delta''_j)_{1,Q_2} \end{matrix} \right] dxdy$$

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$$= \frac{a^{-s/\lambda}\, b^{-t/\mu}}{\lambda\mu}\, \varphi_1\left(-\frac{s}{\lambda}, -\frac{t}{\mu}\right).\theta_2\left(-\frac{s}{\lambda}\right).\theta_3\left(-\frac{t}{\mu}\right) \quad (1.2)$$

provided that

$$-\lambda \min_{1\le j\le M_1}\left[\operatorname{Re}\left(\frac{d_j^{'}}{\delta_j^{'}}\right)\right] < \operatorname{Re}(s) < \lambda \min_{1\le j\le N_1}[\operatorname{Re}\{(1-c_j^{'})/\gamma_j^{'}\}]$$

and

$$-\mu \min_{1\le j\le M_2}\left[\operatorname{Re}\left(\frac{d_j^{''}}{\delta_j^{''}}\right)\right] < \operatorname{Re}(t) < \mu \min_{1\le j\le N_2}[\operatorname{Re}\{(1-c_j^{''})/\gamma_j^{''}\}]$$

The following result can be easily obtained from the result (1.2)

$$\int_0^\infty \dots \int_0^\infty x_1^{\alpha_1-1}\dots x_r^{\alpha_r-1} H^{0,0:\{M_r,N_r\}}_{P,Q:\{P_r,Q_r\}}\left[\begin{matrix} s_1\, x_1 \\ \vdots \\ s_r\, x_r \end{matrix}\left|\begin{matrix} (a_j;\alpha_j^{'},\dots,\alpha_j^{(r)})_{1,P}: & \{(c_j^{(r)},\gamma_j^{(r)})_{1,P_r}\} \\ (b_j;\beta_j^{'},\dots,\beta_j^{(r)})_{1,Q}: & \{(d_j^{(r)},\delta_j^{(r)})_{1,Q_r}\} \end{matrix}\right.\right] dx_1\dots dx_r$$

$$= s_1^{-\alpha_1}\dots s_r^{-\alpha_r}\,\psi(-\alpha_1,\dots,-\alpha_r)\,\varphi_1(-\alpha_1)\dots\varphi_r(-\alpha_r) \quad (1.3)$$

provided that

$$-\min_{1\le j\le M_i}\left[\operatorname{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] < \operatorname{Re}(\alpha_i) < \min_{1\le j\le N_i}[\operatorname{Re}\{(1-c_j^{(i)})/\gamma_j^{(i)}\}]\ \forall\ i=1,2,\dots,r$$

2. USEFUL RESULT

We obtain the following result, which will be required in the next section

$$H\left\{x_1^{\alpha_1}\dots x_r^{\alpha_r}\; H^{o,P+Q:\{m_r+Q_r-M_r,\,n_r+P_r-N_r\}}_{p+P+Q,\,q:\{p_r+P_r,\,q_r+Q_r\}}\left[\begin{matrix} y_1x_1^{t_1} \\ \vdots \\ y_rx_r^{t_r}\end{matrix}\right|\right.$$

$$\left.(g_j;n_j^{(i)},\dots,n_j^{(r)})_{1,p},\left(1-a_j-\sum_{i=1}^r\alpha_j^{(i)}(\alpha_1+1);\alpha_j^{'}t_1,\dots,\alpha_j^{(r)}t_r\right)_{1,P},\left(b_j+\sum_{i=1}^r\beta_j^{(i)}(\alpha_i+1);-\beta_j{'}t_1,\dots,-\beta_j^{(r)}t_r\right)_{1,Q}:R \atop \underline{\qquad\qquad},(h_j;\sigma_j{'},\dots,\sigma_j^{(r)})_{1,q}:S\right]$$

$$; s_1, ..., s_r \} = s_1^{-\alpha_1 - 1} \ldots s_r^{-\alpha_r - 1} H^{0,0:\{m_r, n_r\}}_{p,q:\{p_r, q_r\}} \left[\begin{matrix} y_1 s_1^{-t_1} \\ \vdots \\ y_r s_r^{-t_r} \end{matrix} \middle| \begin{matrix} (g_j; n_j', ..., n_j^{(r)})_{1,p} : \{(k_j^{(r)}, z_j^{(r)})_{1,p_r}\} \\ (h_j; \sigma_j', ..., \sigma_j^{(r)})_{1,q} : \{(\rho_j^{(r)}, \omega_j^{(r)})_{1,q_r}\} \end{matrix} \right] \qquad (2.1)$$

provided that $t_1 > 0, ..., t_r > 0.$

where

$$R = (1 - c_j' - (\alpha_1 + 1)\gamma_j', t_1\gamma_j')_{N_1+1, P_1}, (k_j', \tau_j')_{1,p_1}, (1 - c_j' - (\alpha_1 + 1)\gamma_j', t_1\gamma_j')_{1,N_1}; ...; (1 - c_j^{(r)} - (\alpha_r + 1)\gamma_j^{(r)}, t_r\gamma_j^{(r)})_{1+N_r, P_r},$$

$$(k_j^{(r)}, \tau_j^{(r)})_{1,p_r}, (1 - c_j^{(r)} - (\alpha_r + 1)\gamma_j^{(r)}, t_r\gamma_j^{(r)})_{1,N_r}$$

$$S = (1 - d_j' - (\alpha_1 + 1)\delta_j', t_1\delta_j')_{M_1+1, Q_1}, (\rho_j', \omega_j')_{1,q_1}, (1 - d_j' - (\alpha_1 + 1)\delta_j', t_1\delta_j')_{1,M_1}; ...; (1 - d_j^{(r)} - (\alpha_r + 1)\delta_j^{(r)}, t_r\delta_j^{(r)})_{M_r+1, Q_r},$$

$$(\rho_j^{(r)}, \omega_j^{(r)})_{1,q_r}, (1 - d_j^{(r)} - (\alpha_r + 1)\delta_j^{(r)}, t_r\delta_j^{(r)})_{1,M_r}$$

Proof: The result (2.1) can be easily obtained with the help of result (1.3).

3. MAIN THEOREM

If

(i) $T[s_1, ..., s_r] = H[f(x_1, ..., x_r); s_1, ..., s_r]$

$$= \int_0^\infty \ldots \int_0^\infty H^{0,0:\{m_r, n_r\}}_{p,q:\{p_r, q_r\}} \left[\begin{matrix} s_1 x_1 \\ \vdots \\ s_r x_r \end{matrix} \middle| \begin{matrix} (g_j; n_j', ..., n_j^{(r)})_{1,p} : & \{(k_j^{(r)}, \tau_j^{(r)})_{1,p_r}\} \\ (h_j; \sigma_j', ..., \sigma_j^{(r)})_{1,q} : & \{(\rho_j^{(r)}, \omega_j^{(r)})_{1,q_r}\} \end{matrix} \right] f(x_1, ..., x_r)\, dx_1 \ldots dx_r \qquad (3.1)$$

(ii) $s_1^{-\alpha_1 - 1} \ldots s_r^{-\alpha_r - 1} T[s_q^{-t_1}, ..., s_r^{-t_r}] = H[g(x_1, ..., x_r); s_1, ..., s_r]$

$$= \int_0^\infty \ldots \int_0^\infty H^{0,0:\{M_r, N_r\}}_{P,Q:\{P_r, Q_r\}} \left[\begin{matrix} s_1 x_1 \\ \vdots \\ s_r x_r \end{matrix} \middle| \begin{matrix} (a_j; \alpha_j', ..., \alpha_j^{(r)})_{1,P} : & \{(c_j^{(r)}, \gamma_j^{(r)})_{1,P_r}\} \\ (b_j; \beta_j', ..., \beta_j^{(r)})_{1,Q} : & \{(d_j^{(r)}, \delta_j^{(r)})_{1,Q_r}\} \end{matrix} \right] g(x_1, ..., x_r)\, dx_1 \ldots dx_r \qquad (3.2)$$

and

(iii) $t_1 > 0, ..., t_r > 0$

then $g(x_1, ..., x_r) = x_1^{\alpha_1}, ..., x_r^{\alpha_r} \int_0^\infty \ldots \int_0^\infty H^{0, P+Q:\{m_r+Q_r-M_r, n_r+P_r-N_r\}}_{p+P+Q, q:\{p_r+P_r, q_r+Q_r\}} \left[\begin{matrix} y_1 x_1^{t_1} \\ \vdots \\ y_r x_r^{t_r} \end{matrix} \right|$

$$(g_j; n_j^{(i)}, ..., n_j^{(r)})_{1,p}, \left(1-a_j-\sum_{i=1}^{r}\alpha_j^{(i)}(\alpha_1+1); \alpha_j^{'}t_1, ..., \alpha_j^{(r)}t_r\right)_{1,P}, \left(b_j+\sum_{i=1}^{r}\beta_j^{(i)}(\alpha_i+1); -\beta_j^{'}t_1, ..., -\beta_j^{(r)}t_r\right)_{1,Q} : R \quad \_\_\_\_\_\_\_, (h_j; \sigma_j^{'}, ..., \sigma_j^{(r)})_{1,q} : S \Bigg]$$

$$f(y_1, ..., y_r)\, dy_1 \ldots dy_r \qquad (3.3)$$

provided that the integrals involved in (3.1) to (3.3) are absolutely convergent.

Proof : With the help of (3.1), we obtain

$$s_1^{-\alpha_1-1}, ..., s_r^{-\alpha_r-1}\, T[s_1^{-t_1}, ..., s_r^{-t_r}] = \int_0^{\infty} \int_0^{\infty} s_1^{-\alpha_1-1} ..., s_r^{-\alpha_r-1}.$$

$$H_{p,q:\{p_r,q_r\}}^{0,0:[m_r,n_r]}\left[\begin{array}{c} y_1 s_1^{-t_r} \\ \vdots \\ y_r s_r^{-t_r} \end{array}\middle| \begin{array}{l} (g_j; n_j^{'}, ..., n_j^{(r)})_{1,p} : \{(k_j^{(r)}, z_j^{(r)})_{1,p_r}\} \\ (h_j; \sigma_j^{'}, ..., \sigma_j^{(r)})_{1,q} : \{(\rho_j^{(r)}, \omega_j^{(r)})_{1,q_r}\} \end{array}\right] f(y_1, ..., y_r)\, dy_1 ...dy_r \qquad (3.4)$$

Using (2.1) in (3.4), we get

$$s_1^{-\alpha_1-1} \ldots s_r^{-\alpha_r-1}\, T[s_1^{-t_1}, ..., s_r^{-t_r}] = \int_0^{\infty} \ldots \int_0^{\infty} H\{x_1^{\alpha_1} \ldots x_r^{\alpha_r} \, . \, H_{p+P+Q,q:\{p_r+P_r,q_r+Q_r\}}^{0,P+Q:\{m_r+Q_r-M_r,n_r+P_r-N_r\}}\left[\begin{array}{c} y_1 x_1^{t_1} \\ \vdots \\ y_r x_r^{t_r} \end{array}\right.$$

$$(g_j; n_j^{(i)}, ..., n_j^{(r)})_{1,p}, \left(1-a_j-\sum_{i=1}^{r}\alpha_j^{(i)}(\alpha_1+1); \alpha_j^{'}t_1, ..., \alpha_j^{(r)}t_r\right)_{1,P}, \left(b_j+\sum_{i=1}^{r}\beta_j^{(i)}(\alpha_i+1); -\beta_j^{'}t_1, ..., -\beta_j^{(r)}t_r\right)_{1,Q} : R \quad \_\_\_\_\_\_\_, (h_j; \sigma_j^{'}, ..., \sigma_j^{(r)})_{1,q} : S \Bigg]$$

$$; s_1, ..., s_r\} \ f(y_1, ..., y_r)\, dy_1 \ldots dy_r$$

$$= \int_0^{\infty} \ldots \int_0^{\infty} H_{P,Q:\{P_r,Q_r\}}^{0,0:\{M_r,N_r\}}\left[\begin{array}{c} s_1 x_1 \\ \vdots \\ s_r x_r \end{array}\middle| \begin{array}{l} (a_j; \alpha_j', ..., \alpha_j^{(r)})_{1,P} : \{(c_j^{(r)}, \gamma_j^{(r)})_{1,P_r}\} \\ (b_j; \beta_j', ..., \beta_j^{(r)})_{1,Q} : \{(d_j^{(r)}, \delta_j^{(r)})_{1,Q_r}\} \end{array}\right] x_1^{\alpha_1} \ldots x_r^{\alpha_r}\, H_{p+P+Q,q:\{p_r+P_r,q_r+Q_r\}}^{0,P+Q:\{m_r+Q_r-M_r,n_r+P_r-N_r\}}\left[\begin{array}{c} y_1 x_1^{t_1} \\ \vdots \\ y_r x_r^{t_r} \end{array}\right.$$

$$(g_j; n_j^{(i)}, ..., n_j^{(r)})_{1,p}, \left(1-a_j-\sum_{i=1}^{r}\alpha_j^{(i)}(\alpha_1+1); \alpha_j^{'}t_1, ..., \alpha_j^{(r)}t_r\right)_{1,P}, \left(b_j+\sum_{i=1}^{r}\beta_j^{(i)}(\alpha_i+1); -\beta_j^{'}t_1, ..., -\beta_j^{(r)}t_r\right)_{1,Q} : R \quad \_\_\_\_\_\_\_, (h_j; \sigma_j^{'}, ..., \sigma_j^{(r)})_{1,q} : S \Bigg]$$

$$dx_1 \ldots dx_r\}. f(y_1, ..., y_r)\, dy_1 \ldots dy_r \qquad (3.5)$$

Now change the order of $x_1, ..., x_r$ and $y_1, ..., y_r$ integrals which is permissible under the conditions stated with the theorem, we have

$$s_1^{-\alpha_1-1} \ldots s_r^{-\alpha_r-1}\, T[s_1^{-t_1}, ..., s_r^{-t_r}] = \int_0^{\infty} \ldots \int_0^{\infty} H_{P,Q:\{P_r,Q_r\}}^{0,0:\{M_r,N_r\}}\left[\begin{array}{c} s_1 x_1 \\ \vdots \\ s_r x_r \end{array}\middle| \begin{array}{l} (a_j; \alpha_j^{'}, ..., \alpha_j^{(r)})_{1,P} : \{(c_j^{(r)}, \gamma_j^{(r)})_{1,P_1}\} \\ (b_j; \beta_j^{'}, ..., \beta_j^{(r)})_{1,Q} : \{(d_j^{(r)}, \delta_j^{(r)})_{1,Q_r}\} \end{array}\right.$$

$$\left\{\int_0^\infty \cdots \int_0^\infty x_1^{\alpha_1} \cdots x_r^{\alpha_r} H^{0,P+Q:\{m_r+Q_r-M_r, n_r+P_r-N_r\}}_{p+P+Q, q:\{p_r+P_r, q_r+Q_r\}} \left[\begin{array}{c} y_1 x_1^{t_1} \\ \vdots \\ y_r x_r^{t_r} \end{array}\right| \right.$$

$$\left. (g_j; n_j^{(i)}, ..., n_j^{(r)})_{1,p}, \left(1-a_j-\sum_{i=1}^{r}\alpha_j^{(i)}(\alpha_1+1); \alpha_j' t_1, ..., \alpha_j^{(r)} t_r\right)_{1,P}, \left(b_j+\sum_{i=1}^{r}\beta_j^{(i)}(\alpha_i+1); -\beta_j' t_1, ..., -\beta_j^{(r)} t_r\right)_{1,Q} : R \atop \_\_\_\_\_\_\_, (h_j; \sigma_j', ..., \sigma_j^{(r)})_{1,q} : S \right]$$

$$f(y_1, ..., y_r)\, dy_1 \dots dy_r \}\, dx_1 \dots dx_r \tag{3.6}$$

Now comparing (3.6) with (3.2), we arrive at the desired result (3.3).

4. SPECIAL CASES

(i) On taking $p, q, P, Q = 0,\ N_1, ..., N_r = 0,\ m_i = m_i + 1,\ n_i = 0,\ p_i = m_i,$

$q_i = m_i + 1, k_j^{(i)} = k_j^{(i)} + \rho_j^{(i)} (j = 1, ..., m_i)\ \forall (i = 1, ..., r), \tau_j' = \omega_j' = 1\ (j = 1, ..., m_i; i = 1, ..., m_1 + 1), ..., \tau_j^{(r)} = \omega_j^{(r)} = 1$

$(j = 1, ..., m_r; i = 1, ..., m_r + 1),\ \ \rho'_{m_1+1} = n', ..., \rho^{(r)}_{m_r+1} = n^{(r)}$ and $M_i = M_i + 1,\ P_i = M_i, Q_i = M_i + 1, C_j^{(i)} = C_j^{(ii)} + d_j^{(i)}$

$(j = 1, ..., M_i)\ \forall\ (i = 1, ..., r), \gamma_j' = \delta_j' = 1\ (j = 1, ..., M_i, i = 1, ..., M_1 + 1), ...\ \gamma_j^{(r)} = \delta_j^{(r)} = 1 (j = 1, ..., M_r; i = 1, ..., M_r + 1),$

$d'_{M_1+1} = \rho', ..., d^{(r)}_{M_r+1} = \rho^{(r)}$ in main theorem, we arrive at the following result :

Theorem 1 If

(i) $T[s_1, ..., s_r] = G[f(x_1, ..., x_r); s_1, ..., s_r]$

$$= \int_0^\infty \cdots \int_0^\infty G^{0,0:\{m_r+1,0\}}_{0,0:\{m_r, m_r+1\}} \left[\begin{array}{c} s_1 x_1 \\ \vdots \\ s_r x_r \end{array}\right| \begin{array}{l} -:(K_j' + \rho_j')_{1,m_1}; ...;\ (K_j^{(r)} + \rho_j^{(r)})_{1,m_r} \\ -:(\rho_j')_{1,m_1,n}; ...;\ \ (\rho_j^{(r)})_{1,m_r,n^{(r)}} \end{array} \right] f(x_1, ..., x_r)\, dx_1 \dots dx_r \tag{4.1}$$

(ii) $s_1^{-\alpha_1 - 1} \dots s_r^{-\alpha_r - 1}\, T[s_1^{-t_1}, ..., s_r^{-t_r}] = G[g(x_1, ..., x_r); s_1, ..., s_r]$

$$= \int_0^\infty \cdots \int_0^\infty G^{0,0:\{M_1+1,0\}}_{0,0:\{M_r, M_r+1\}} \left[\begin{array}{c} s_1 x_1 \\ \vdots \\ s_r x_r \end{array}\right| \begin{array}{l} -:(c_j' + d_j')_{1,M_1}; ...;\ \ (c_j^{(r)} + d_j^{(r)})_{1,M_r} \\ -:(d_j')_{1,M_1,\rho'}; ...;\ \ (d_j^{(r)})_{1,M_r},\ \rho^{(r)} \end{array} \right] g(x_1, ..., x_r)\, dx_1 \dots dx_r \tag{4.2}$$

and

$$\mathrm{Re}[\alpha_1 + 1 + t_1 n' + \rho' + t_1 \rho_j'\ (j' = 1, ..., m_1) + d_j' (j = 1, ..., m_1)] > 0, ...,$$

$$\mathrm{Re}[\alpha_r + 1 + t_r n^{(r)} + \rho^{(r)} + t_r \rho_j^{(r)} \ (j' = 1, ..., m_r) + d_j^{(r)} (j' = 1, ..., m_r)] > 0,$$

$$\mathrm{Re}(s_1, ..., s_r) > 0,\ 0 < t_1 \le 1, ..., 0 < t_r \le 1$$

then $\quad g(x_1, ..., x_r) = x_1^{\alpha_1}, ..., x_r^{\alpha_r}$

$$\int_0^\infty .... \int_0^\infty H^{0,0:\{m_1+1, m_r\}}_{0,0:\{m_r+M_r, m_r+M_r+2\}} \left[\begin{array}{c} y_1 x_1^{t_1} \\ \vdots \\ y_r x_r^{t_r} \end{array} \middle| \begin{array}{l} \text{——} : (-c'_j - d'_j - \alpha_1, t_1)_{1, M_1}, (k'_j + \rho'_j, 1)_{1, m_1}; ...; (-c_j^{(r)} - d_j^{(r)} - \alpha_r, t_r)_{1, M_r}, (K_j^{(r)} + \rho_j^{(r)}, 1)_{1, m_r} \\ \text{——} : (\rho'_j, 1)_{1, m_1}, (n', 1), (-d'_j - \alpha_1, t_1)_{1, M_1}, (-\rho' - \alpha_1, t_1); ...; (\rho_j^{(r)}, 1)_{1, m_r}, (n^{(r)}, 1), (-d_j^{(r)} - \alpha_r, t_r)_{1, M_r}, (-\rho^{(r)} - \alpha_r, t_r) \end{array} \right] f(y_1, ..., y_r)\, dy_1, ..., dy_r \quad (4.3)$$

provided that the integrals involved in (4.1) to (4.3) are absolutely convergent.

(ii) $m_1 = 0, ..., m_r = 0,\ \rho' = \rho^{(r)} = n' = n^{(r)} = 0$ and $M_1 = 1, ..., M_r = 0$ in theorem 1, we find the following result involving the multivariable laplace transform :

Theorem 2 If

(i) $T[s_1, ..., s_r] = L\{f(x_1, ..., x_r\}; s_1, ..., s_r\}$

$$= \int_0^\infty ... \int_0^\infty e^{-s_1 x_1 - .... - s_r x_r} f(x_1, ..., x_r)\, dx_1 ... dx_r \quad (4.4)$$

(ii) $s_1^{-\alpha_1 - 1} ... s_r^{-\alpha_r - 1} T[s_1^{-t_1}, ..., s_r^{t_r}] = L\{g(x_1, ..., x_r); s_1, ..., s_r\}$

$$= \int_0^\infty ... \int_0^\infty e^{-s_1 x_1 - ... - s_r x_r} g(x_1, ..., x_r)\, dx_1 ... dx_r \quad (4.5)$$

and

(iii) $\mathrm{Re}[\alpha_1 + 1] > 0, ..., \mathrm{Re}[\alpha_r + 1] > 0, \mathrm{Re}[s_1, ..., s_r] > 0, 0 < t, \le 1, \ ..., 0 < t_r \le 1$

then $g(x_1, ..., x_r) = x_1^{\alpha_1} ... x_r^{\alpha_r} \int_0^\infty ... \int_0^\infty J^t_{\alpha_1}(y_1 x^{t_1}) ... J^{t_r}_{\alpha_r}(y_r x^{t_r})\ f(y_1, ..., y_r)\, dy_1, ..., dy_r$ (4.6) provided that the integrals involved in (4.4) to (4.6) are absolutely convergent.

The functions $J_{\alpha_1}^{t_1}(y_1 x^{t_1}), \ldots, J_{\alpha_r}^{t_r}(y_r x^{t_r})$ occurring in (4.6) are Maitland's generalized Bessel functions which are the special cases of H-function.

(iii) Putting $r=2$ in main theorem, we get the theorem obtained by V.B.L. Chaurasia and Neeti Gupta [4].

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